AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

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No. 849.

SOME EXPERIMENTS ON BRIDGES UNDER MOVING TRAIN-LOADS.

By F. E. TURNEAURE, Assoc. Am. Soc. C. E.

PRESENTED DECEMBER 7TH, 1898.

WITH DISCUSSION.

INTRODUCTORY.

All specifications for railroad bridges recognize in some way the principle that more liberal provision should be made for the stresses due to moving loads than for those due to fixed loads. In some specifications the method of treatment appears to be based entirely upon the theory of the fatigue of metals, and the intensity of stress is determined by a formula of the form: $p = a\left(1 + \frac{\text{Min.}}{\text{Max.}}\right)$. In others the element of fatigue is not considered, but the live-load stresses are increased by a certain percentage "for impact," and then treated as dead-load stresses, thus assuming that the only difference between the effects of live and dead loads arises from the fact that the actual stresses produced by moving loads are greater than those produced by the same loads when stationary. Still other specifications take account of both fatigue and impact.

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While all engineers are agreed that some extra allowance must be made for live loads, yet there exists the widest difference of opinion as to how much this allowance should be and how it should be distributed among the different members of the bridge. It is certainly logical that if both fatigue and impact exist, both should be provided for, although, perhaps, by a single formula; but before such a formula can be determined the two effects must be considered separately. Considerable agreement has been reached in the matter of fatigue formulas, but, owing to the lack of reliable data, impact formulas are, at the present time, largely a matter of guess work.

In the hope of contributing something to the solution of this problem, the author, during the summer of 1897, carried out a series of experiments on several bridges of widely varying span and design; and it is the purpose of this paper to present the results of these tests, together with a description of the apparatus used, and a discussion of the points involved.

Apparatus.

The instruments used were three in number and of two kinds, viz., one instrument for measuring the deflection of the structure as a whole, and two instruments for measuring extensions and compressions of individual members.

The Deflectometer.—The instrument used for measuring deflections was a Fraenkel deflectometer,* the design of the late Professor Fraenkel of Dresden, Germany. Plate IX is a photograph of this instrument showing it attached as it would be in actual use on a plate girder. In this type of instrument the main part of the apparatus is fastened to the bridge, and connection is made by means of a wire, to a heavy weight placed on the ground directly underneath. This wire is made fast, through an interposed steel ribbon, to the circumference of a thick disk, or wheel, mounted on the framework, which disk is cast in one part with another of twice its diameter and having the same axis. On the larger disk is fastened a second steel ribbon attached at its other end to the circumference of a spirally shaped, hollow wheel containing a clock spring By means of a crank the spring may be wound up so as to produce in the steel ribbon a tension sufficient to hold the vertical wire taut, while at the same time allowing free movement of the wire

*Cuts of a slightly different form of this instrument are shown in *Der Civilingenieur*, 1884, Plate xxx; also in *Engineering News*, February 8th, 1894, p. 115.

relative to the frame of the instrument. The recording pencil is attached to the short steel ribbon previously mentioned, and it has, therefore, a motion relative to the frame of the apparatus twice as great as the motion of the bridge. A spindle is provided for holding a roll of paper; another one upon which the paper is wound, and a drum over which the paper passes, and along which the pencil moves as the bridge moves up and down. The paper is moved by electrically controlled clockwork placed inside the drum. The governor of the clockwork was so regulated as to make the rate of motion of the paper about $\frac{1}{10}$ in. per second. The apparatus is very compact, and can be fastened by means of heavy, pointed screws to any horizontally projecting plate, or to a flange of a bridge post. A second pencil was put on this instrument for the purpose of recording electrically the passage of the wheels of the trains, but the operating magnets did not work satisfactorily, and the desired information was obtained mainly from similar pencils attached to the other instruments.*

The Extensometers.—The instruments used for measuring strains in the members were also of a form invented by Professor Fraenkel, but greatly improved by Oscar Leuner, of Dresden, the maker of all the Fraenkel instruments.†

Plate X is a photograph of the recording part of one of the instruments used. In this form of extensometer the longitudinal distortion of a certain length of a member is multiplied about 140 times by means of a compound lever, the motion being communicated to a pencil which traces a line on a moving strip of paper, as in the deflectometer. Although this instrument was designed some seventeen years ago, yet it is only within the last two or three years that it has been so constructed as to reduce the friction of the rapidly moving parts to such an extent as to make it sufficiently sensitive for good work. As now constructed,

* Another deflectometer of this type is that invented and used by M. Rabut. It is illustrated in *Annales des Ponts et Chaussées* of 1896, Vol. xii., p. 381, in a very complete article, in which this and other instruments are described, and their use in the testing of bridges discussed. In this instrument the wire is held by a long coiled spring attached to a separate point of the bridge, and the relative motion is imparted to the recording pencil by attaching the wire to the short end of a lever, the pencil being at the other end. The lever is made very light, and, on the whole, the arrangement is probably better, although not so convenient, as that of the Fraenkel apparatus. By an ingenious arrangement of two weights, and two wires which intersect near the recording apparatus, M. Rabut is enabled to record lateral as well as vertical movements.

For descriptions of examples of the other type of deflectometer—that in which the recording parts are supported from the ground and the pencils are attached to the structure—see "Vibration of Bridges," by S. W. Robinson, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. xvi, 1887, p. 42; and "Bridge Deflections," by M. A. Howe, M. Am. Soc. C. E., *Journal of the Association of Engineering Societies*, Vol. xiv, 1895, p. 513.

† For a description and illustration of the original form of instrument, see *Der Civilingenieur*, 1881, p. 249.

PLATE IX.
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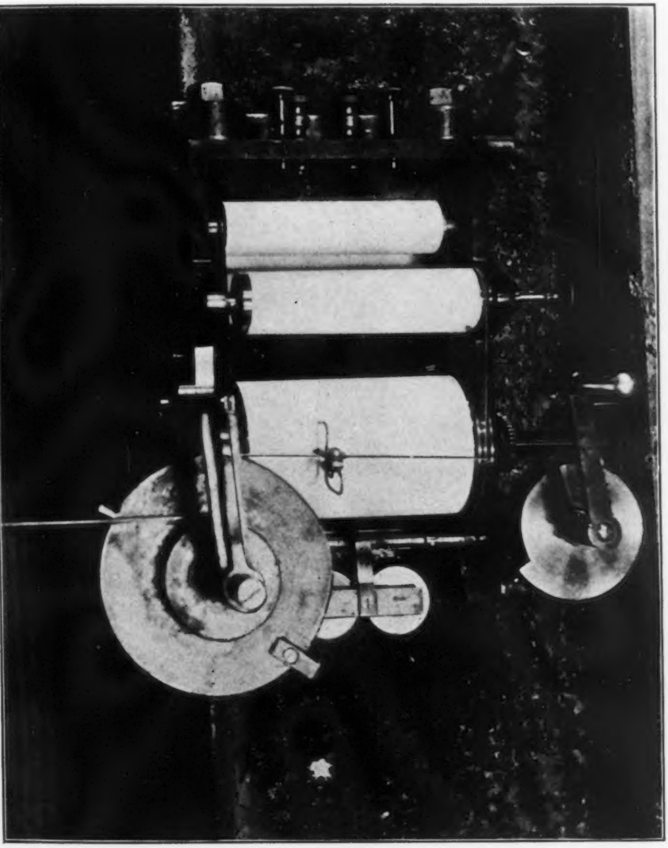
TURNEAURE ON MOVING TRAIN-LOAD EXPERIMENTS.



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all bearings are knife-edge, and all connections are made by means of steel ribbons attached to cylindrical surfaces. The levers are mounted in a suitable framework, which carries also the clockwork mechanism for moving the paper, and which is provided with screws for attaching the apparatus to the bridge member. The recording pencil is held by a light carriage running parallel to the axis of the drum on which the paper is stretched. This carriage is moved by two small steel ribbons attached to a sector at the end of the lever system, which arrangement gives rectilineal motion to the pencil. To the short end of the lever system is fastened one end of a long, light rod, clamped at its other end to the bridge member. In the instruments as used, these rods were of such a length that, when properly adjusted, the apparatus would measure the distortion of one meter's length of the bridge member. The multiplication of the distortion of the bridge member was such that, assuming the modulus of elasticity to be 29 000 000 lbs. per square inch, 1 in. on the diagram corresponds to a stress of 5 600 lbs. per square inch, which value has been used in the computations. Additional pencils for recording the passage of wheels were also attached to these instruments, and in this case they worked satisfactorily.

In order to make the friction of the pencil on the paper as small as possible, metallic paper and aniline pencils were used, these being furnished with the instruments. With this arrangement a distinct mark is made with an exceedingly light pressure, and the pencil does not wear smooth as lead does. The speed of the paper in the extensometers was, in one case, about $\frac{3}{10}$, and in the other case, about $\frac{5}{10}$, in. per second. The speed was determined at frequent intervals, and was fairly constant; but, owing to the method of governing the clockwork, it could not easily be regulated to any particular value. The speeds of trains were determined from the records of the additional pencils above mentioned, the speed of the paper and the length of the locomotive wheel-base being known.*

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* Another extensometer which has been used to a considerable extent in France is that known as the Manet apparatus. It has been perfected by M. Rabut, and used quite extensively by him. The instrument is not a self-recording one, the motion being multiplied and communicated to a pointer moving over a dial. It is illustrated and described in the article by M. Rabut, before mentioned; also, in Johnson's "Framed Structures," p. 230.

Another instrument which should be mentioned in this connection is that constructed by Messrs. J. J. Hankenson and Wm. H. Ledger, and used by them in carrying out a series of tests at Cornell University. In this instrument a much less multiplication is used than in either of the others. Single levers are used, and a continuous record obtained. *Engineering News*, Vol. xxxiii, 1895, p. 300.

Track Instrument.—The electric track switch used was a rather crude affair, but answered the purpose very well. Some difficulty was experienced in devising an apparatus which would work slowly enough under fast trains. That which was finally successful was made as follows: On a long wooden block a saw-blade was placed and loosely held at its two ends under thin metal straps. This blade was then sprung upward and supported on brass springs along the center, and so arranged that on being depressed about ½ in. an electric circuit would be closed. This contrivance was fastened just outside the rail and parallel thereto, and at such an elevation that a passing wheel would keep the circuit closed while moving a distance of 12 or 15 ins.

General Description of the Experiments.

The experiments in question consisted in obtaining records of stress and deflection under moving trains from a number of bridges on the Chicago, Milwaukee and St. Paul, and the Union Pacific railroads. Tests were made on twelve plate-girder bridges, of spans varying from 25 to 80 ft. in length, and on eleven truss bridges of spans from 100 to 200 ft. in length.

The chief characteristics of these bridges are given in Tables Nos. 1 and 2. All the plate girders are through bridges, and, with the exception of Nos. 10 and 12, the ties are supported on shelf angles. They are, no doubt, somewhat shallower than deck girders of like span, and, therefore, somewhat more flexible. All the girders, however, are of recent construction and represent modern practice. All the truss bridges are of the Pratt type, except Nos. 20 and 21, which are Pegram trusses. The dates of erection indicate fairly well the character of the construction, Nos. 13 and 15 being light structures; while Nos. 14, 17, 18 and 19, are fairly heavy, modern, Pratt trusses. The two Pegram trusses have long panels and a suspended floor system with plate hangers. The last four bridges are on the Union Pacific, all others are on the Chicago, Milwaukee and St. Paul Railway.

In the testing of girder bridges, the deflectometer was usually fastened to a flange at the center of the bridge, and one or both of the extensometers attached to the flange of the girder as near the center as possible. For the sake of comparison, simultaneous measurements of stress were frequently made, in some cases on the two lower flanges, and in others on the outside lower and outside upper flanges. In gen-

PLATE X.
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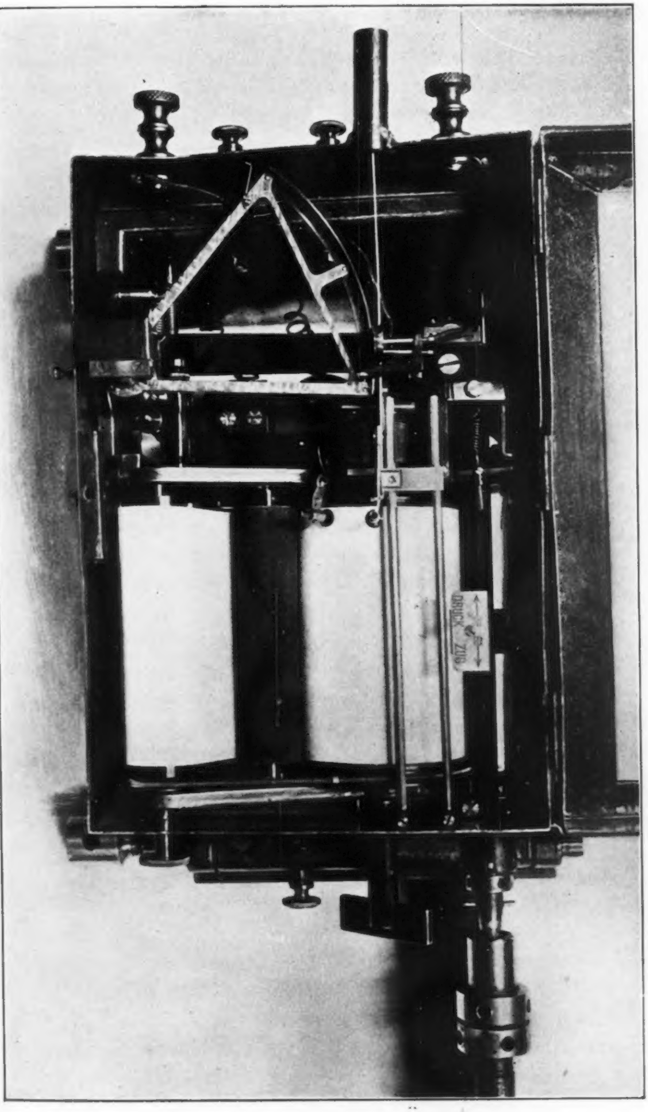

eral, about twelve or fifteen experiments were made on each girder bridge, this being usually a sufficient number to bring out as great a variety of speeds and locomotive effects as would be likely to occur with any frequency. In most of the girders the effect of the locomotives was soon seen to be the feature of chief interest, that of either passenger or freight cars not being of special importance.

In the experiments on trusses the deflectometer was usually attached to the post nearest the center of the bridge, and records were obtained from it in that position for all tests on the structure. At the same time the extensometers were attached to various members of the bridge. After a little experience it was concluded that the members from which the most significant curves could be obtained were: A lower chord member, the main diagonal nearest the center, and the hip vertical; and these were, therefore, the members usually experimented upon. By the use of both extensometers determinations were made, in many cases, of the bending in individual bars and of the relative tension in the various bars of the same member. Such experiments, however, were made of secondary importance, the main object of the work being to secure a reasonable number of diagrams from as many different structures as possible. As a rule, from thirty to forty experiments were made on each truss.

In all, about four hundred experiments were made, three curves usually being obtained from each. With few exceptions, the experiments were made with the trains of the regular traffic. As in most cases there were but ten or twelve movements during the day, not as many diagrams were obtained as might have been desirable; but it was thought that the work would be more valuable if a number of structures of different span-lengths were tested than if a great number of experiments were made on two or three bridges. This idea was strengthened by the fact that in those places where a longer stay was made at a structure, very little which was new was observed in the deflection curves after the first day's work.

On two different occasions, through the kindness of the railroad officials, a short freight train was placed at the disposal of the experimenters. In one case the train consisted of an American locomotive, one lightly loaded car, and three empties; and with this, Tests Nos. 97 to 107 were made on Bridge No. 14. In the other case it consisted of a ten-wheel engine, four heavily loaded cars, and a caboose; and with this

train tests were made on Bridges Nos. 11, 12, 18 and 19. The use of a special train in making such tests is obviously of great advantage, both in enabling the tests to be made very rapidly, and in facilitating comparisons. For the tests with special trains, and for many of the others, stress curves were computed, the data regarding weights of locomotives and cars being furnished by the railroad companies. In the computations the influence-line method was used as being the only practicable one for such a large amount of work. The results of the experiments are presented and discussed further on.

Working of the Apparatus.

The working of the apparatus was in general quite satisfactory. The deflectometer proved very convenient to adjust, and was very reliable in its working, except for the short-girder spans. With spans less than about 50 ft. in length, the jarring of the girder agreed so closely in period with that of the moving parts of the instrument (about 20 per second) that the apparatus was set into excessive vibration, and the resulting records in such cases are not of much value. Some reliable curves, however, were obtained from a 25-ft. span by placing the instrument on the ground and making the connection by means of gas pipe. These curves show that the vibration or jarring of the bridge at this rapid rate was really of very small amplitude, a very small fraction of the disturbances of a longer period which are discussed further on. In the longer girders and in the truss bridges this jarring action nearly disappeared, and the curves obtained from the deflectometer bring out very clearly the vibration of the structures without interference by instrumental vibrations. If the moving parts of the instrument were made much lighter, it is believed that it would be much more useful for testing very short spans.

The moving parts of the extensometers, owing to the high degree of multiplication of the instruments, constitute a rather flexible system, and have, therefore, a rather slow natural rate of vibration, much slower than the deflectometer. This rate of vibration was found experimentally to be about 10 per second, and, therefore, where the variation in stress about equals or exceeds this in rapidity, the records of the instruments are not reliable. Since the rates of vibration of the structures, or of individual members, were in most cases much less than this, little trouble was experienced on this score. The jarring action

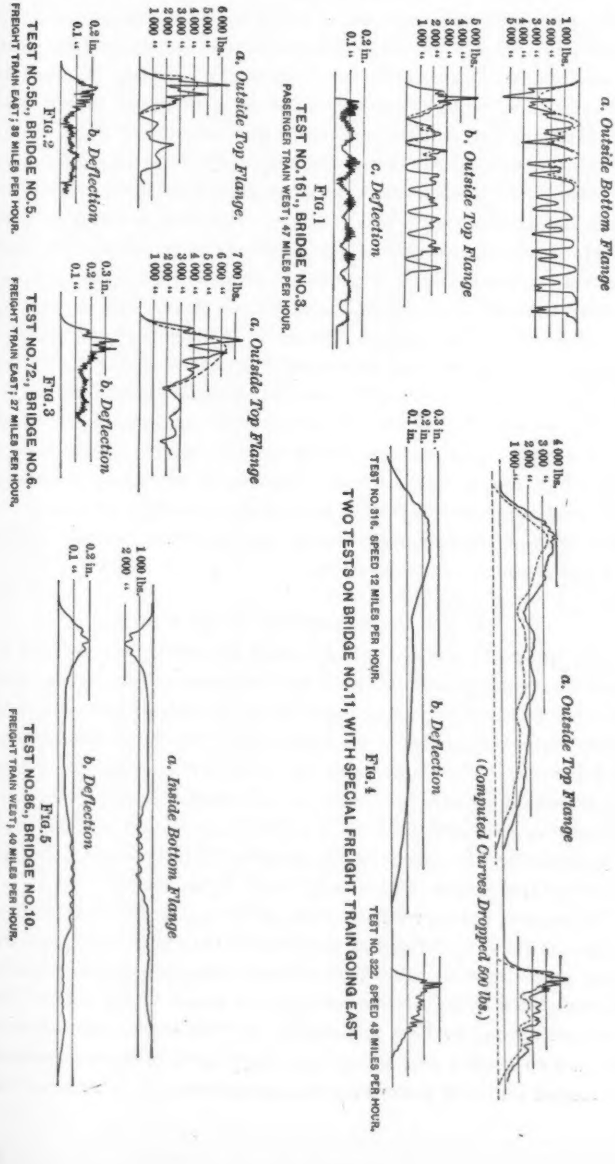

FIG. 1
TEST NO.161., BRIDGE NO.3.
PASSENGER TRAIN WEST; 47 MILES PER HOUR.

FIG. 2
TEST NO.55., BRIDGE NO.5.
FREIGHT TRAIN EAST; 39 MILES PER HOUR.

FIG. 3
TEST NO.72., BRIDGE NO.6.
FREIGHT TRAIN EAST; 27 MILES PER HOUR.

FIG. 4
TEST NO. 316. SPEED 12 MILES PER HOUR.
TEST NO. 322. SPEED 48 MILES PER HOUR.
TWO TESTS ON BRIDGE NO.11, WITH SPECIAL FREIGHT TRAIN GOING EAST

FIG. 5
TEST NO.86., BRIDGE NO.10.
FREIGHT TRAIN WEST; 40 MILES PER HOUR.

which interfered with the working of the deflectometer appeared to be much too rapid or too small to affect the extensometers, as these instruments invariably gave curves free from such vibrations. The fact that the rates of vibration of the two kinds of instruments were so much different furnished a valuable check to the workings of both.

In order to make the rate of vibration of the extensometers as rapid as possible, the steel carriages supporting the pencils were replaced by much simpler ones made of aluminum. It would, however, be desirable to reduce the weights of the rapidly moving parts still further. The author does not believe it possible with any form of apparatus yet devised to record correctly all the changes in stress which occur in certain members; as, for example, flanges of short girders and stringers, and possibly main members of trusses. From observations on deflection it is believed that these rapid changes, accompanied by a jarring sensation, are very small; but they can only be measured by an instrument whose moving parts are almost without weight. A reference to some of the curves obtained in the experiments of Messrs. Hankenson and Ledger, referred to further on, will show several places where this same effect of instrumental vibration has, doubtless, entirely masked the real distortion in the members.

General Characteristics of the Curves.

On pages 417, 419 and 421, are given several curves showing the main characteristics of the principal diagrams taken. Ordinates on the deflection curves indicate deflections in inches; ordinates on the stress curves indicate stress per square inch. The dotted lines are computed curves. The diagrams shown on page 417 are from plate girders, while those on pages 419 and 421 are from trusses. Figs. 1, 2 and 3, were taken from short girders of 30, 35 and 45-ft. span-lengths, respectively; Fig. 4 from a 52-ft. span, and Fig. 5 from a 70-ft. through girder. The details of the various bridges are given in Table No. 1.

In general, the curves from plate girders taken from rapidly moving trains show, in the part of the diagram caused by the locomotive, large projections, or vibrations, usually corresponding in period to the revolution of the drivers, and due no doubt to the action of the counterweights (see Figs. 1, 2 and 3). On the shorter spans this action was so sudden and severe that the pencils of the extensometers were often evidently thrown too far, as can be seen by comparison with

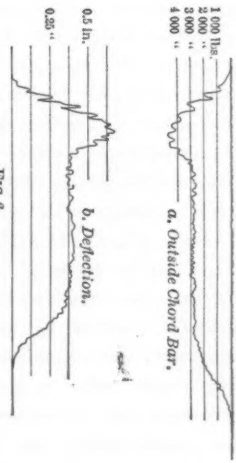

Fig. 6.
TEST NO. 13., BRIDGE NO. 13.
PASSENGER TRAIN WEST; 40 MILES PER HOUR.

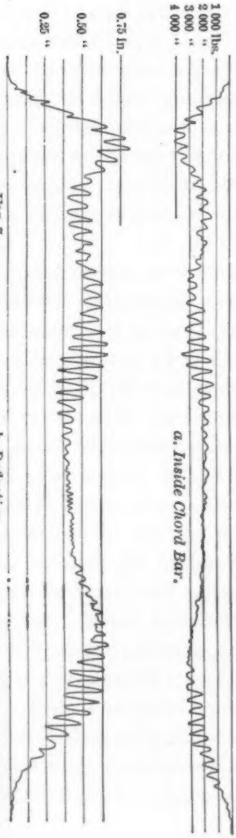

Fig. 7.
TEST NO. 14., BRIDGE NO. 13.
FREIGHT TRAIN EAST; 28 MILES PER HOUR.

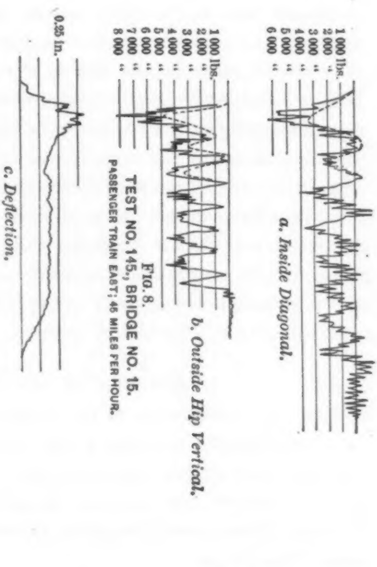

Fig. 8.
TEST NO. 145., BRIDGE NO. 15.
PASSENGER TRAIN EAST; 45 MILES PER HOUR.

the deflections (Fig. 2). For the longer spans, as in Fig. 5, this effect of drivers diminished very much and the curves from the two instruments agreed better. Where the speeds were less than 15 or 20 miles per hour, the vibrations were small. In Figs. 1, 2 and 3, the deflection curves show the effect of the jarring action before mentioned, and in Fig. 1 the vibrations are undoubtedly much exaggerated by the instrument. As a rule, the larger the bridge, the less this effect, and in the case of the two girder spans of the through type it did not appear. Passenger cars produced no noticeable vibrations in girders other than this jarring action, and the same is true of freight cars on all the girders except the two through spans (see Fig. 5).

Specimens of the diagrams obtained from truss spans are shown in Figs. 6 to 11 inclusive. In very many cases considerable vibration of the truss was caused by the locomotive, as shown by the deflection diagrams in nearly all the figures. This was true for passenger as well as for freight locomotives, and for a considerable range of speed, although not usually caused by trains running slower than 20 miles per hour. Passenger cars never caused large vibrations, nor did empty freight cars, but loaded freight cars often did, as in Fig. 7. Diagrams of chord stress are seen to be quite similar to the corresponding deflection curves. Those for stress in the diagonal and in the hip vertical show, of course, large variations due to the wheel-load concentrations, and, besides, exhibit very large vibrations under the locomotive, which correspond usually to the vibrations in the deflection curves. The rapid vibrations shown in the stress curves for the diagonals in Figs. 8, 9 and 10 are due to the vibration of the individual bars. These are much more rapid than those caused by the vibration of the bridge as a whole. The agreement between the computed and the observed stresses varies; but in general the curves from the instruments are similar to the computed curves, although the absolute values of the ordinates differ quite materially on account of secondary stresses.

Discussion of the Experiments.

From an examination of the curves and a consideration of the elements affecting the problem, it was thought that the discussion could best be divided into the following parts: Effects of Speed Alone; Vibrations in Girders and Trusses; Relation of Stresses to Deflections, Secondary Stresses and Computed Deflections; Results of Other Experiments; Conclusions.

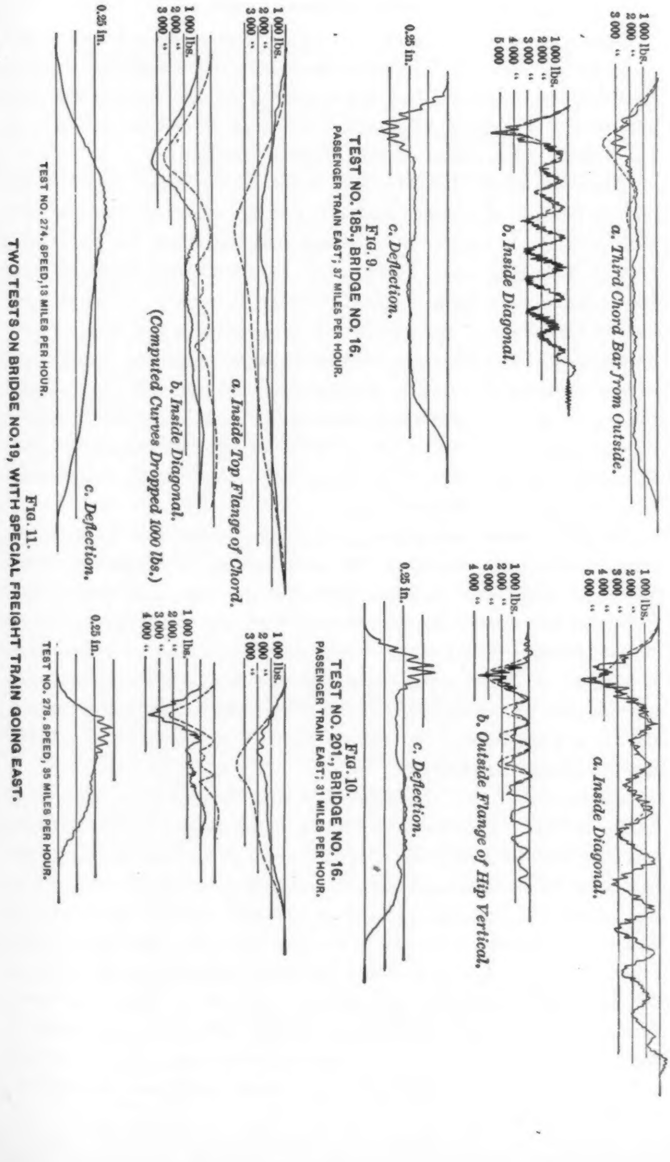

FIG. 9.
TEST NO. 185., BRIDGE NO. 16.
PASSENGER TRAIN EAST; 37 MILES PER HOUR.

FIG. 10.
TEST NO. 201., BRIDGE NO. 16.
PASSENGER TRAIN EAST; 31 MILES PER HOUR.

FIG. 11.
TWO TESTS ON BRIDGE NO. 19, WITH SPECIAL FREIGHT TRAIN GOING EAST.

Effects of Speed Alone.

Inasmuch as in discussions on impact, greater stress is usually laid upon the feature of the rapidity of application of the load than upon vibrations, and, as impact formulas appear to give emphasis in this same direction, it was thought desirable to discover, if possible, the effects of speed alone, when separated from those of vibration.

Experimental Results.—If a train moves over a bridge on a perfectly smooth track at any given speed, and if all the wheels are truly circular and perfectly balanced, it is evident that there will be produced a smooth deflection curve, which, when compared with the curve made by the same train when crossing at a very slow speed, will bring out clearly the effects of the rapidity of application of the load. In the actual case various elements conspire to cause vibrations, which are, if of any considerable duration, of equal amplitude each side of the imaginary smooth curve previously mentioned. From these considerations it was concluded that curves constructed by drawing middle lines through the sinuous lines of the diagrams, could be fairly compared in studying the effects of speed alone.

In making these comparisons, two methods were used. One was to measure the mean ordinate to the curve of the extensometer at some particular point and then compare with the computed stress. The other was to compare the curves from both the extensometer and deflectometer caused by the same train at different rates of speed. The latter is by far the more reliable and satisfactory, as it eliminates many uncertainties in the matter of weights and dimensions, but it requires the use of a special train for testing, which was had only for the few bridges already mentioned.

The results of this comparison for eight bridges are given in Figs. 12, 13, and 14. In all cases the stress in the flange, or other member, was computed for that position of the locomotive which would give the maximum stress. Mean ordinates were then measured to the corresponding points on the diagrams, and these measured stresses then compared with the computed. The ratios of the observed to the computed stresses are plotted in the figures as points, ordinates representing the values of the ratios, and abscissas representing velocities in miles per hour. Dotted horizontal lines representing average ratios have been drawn in those cases where the points could not readily be connected. It was not to be expected that the observed stresses would agree very closely with

COMPARISON OF MEAN STRESSES AND DEFLECTIONS FOR VARIOUS SPEEDS.
PLATE GIRDERS.

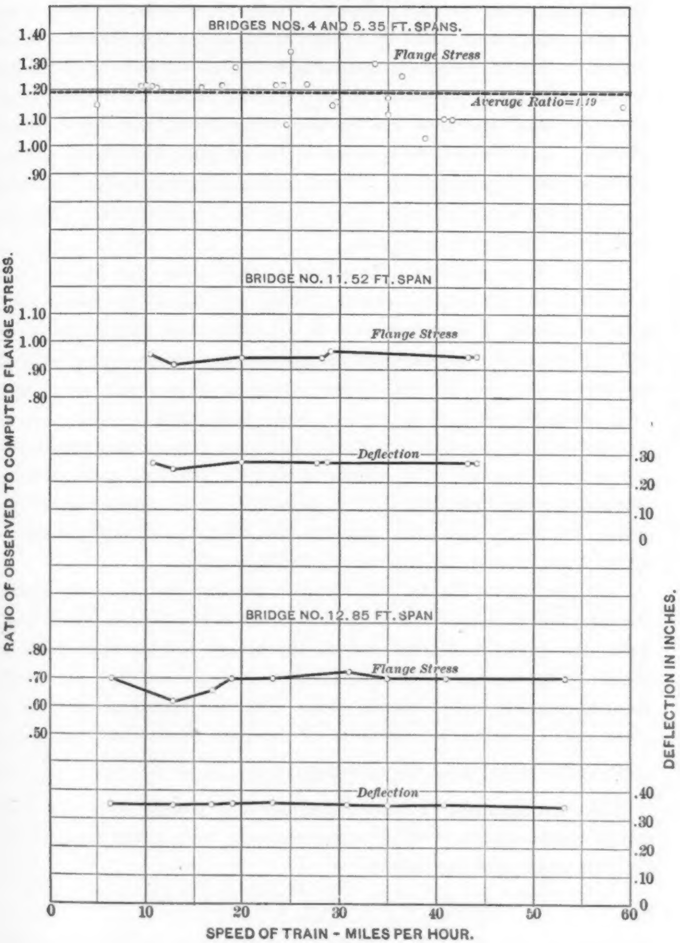

FIG. 12.

the computed, on account of the secondary stresses; but by plotting the observations in this way it was thought that any possible effect of speed would be noticed. In the tests on Bridges Nos. 4 and 5 (Fig. 12) and 16 (Fig. 13) the trains were all different. In the other cases the observations for each bridge were made by a single train, and, although ratios of observed to computed stresses are plotted here also, the computed stresses are the same for all speeds, and the observed values are directly comparable. The deflections are likewise comparable, and are therefore plotted. As to camber, the practice of the Chicago, Milwaukee and St. Paul Railway Company is to omit all camber in girders and to put about the usual amount in trusses. There were no irregularities in the track sufficiently great to be noticeable.

As was to be expected, the points in the diagrams for Bridges Nos. 4, 5 and 16 are somewhat scattered, but the points in all the others show much regularity in arrangement. It is quite evident that the diagrams fail to show in any case any general increase in stress or deflection with increase in speed. On the contrary, they show in nearly every case very constant values at all speeds, and, considering the uniformity of the results and the conditions under which the experiments were made, the conclusion seems to be warranted that the effect of speed alone on bridges of spans exceeding 40 ft. in length is of no consequence whatever. For shorter spans the experiments are not sufficiently accurate to be of much value in a discussion of this question.

Deductions from Theory.—If a load in passing over a bridge causes the structure to deflect so that the curvature of the track is concave upward, the pressure of the load on the bridge will be more than its weight by the amount of the centrifugal force developed. If v = velocity along the curved path (assumed equal to the horizontal velocity); M = mass; and r = radius of curvature of the path of the load (not necessarily of the track), then the centrifugal force, $F, = \frac{Mv^2}{r}$.

Take, for example, the case of a single load moving over a beam of constant cross-section. When the movement is slow, the equation of the path of the moving body is:

$$y = y_o \left(1 - \frac{x}{a}\right)^2 \left(1 + \frac{x}{a}\right)^2,$$

where x and y are the usual coordinates, with origin at center; a = half span; and y_o = central deflection with load at center. The sharpest curvature of this path is at the center, where the radius of curvature,

COMPARISON OF MEAN STRESSES AND DEFLECTIONS FOR VARIOUS SPEEDS.
TRUSSES.

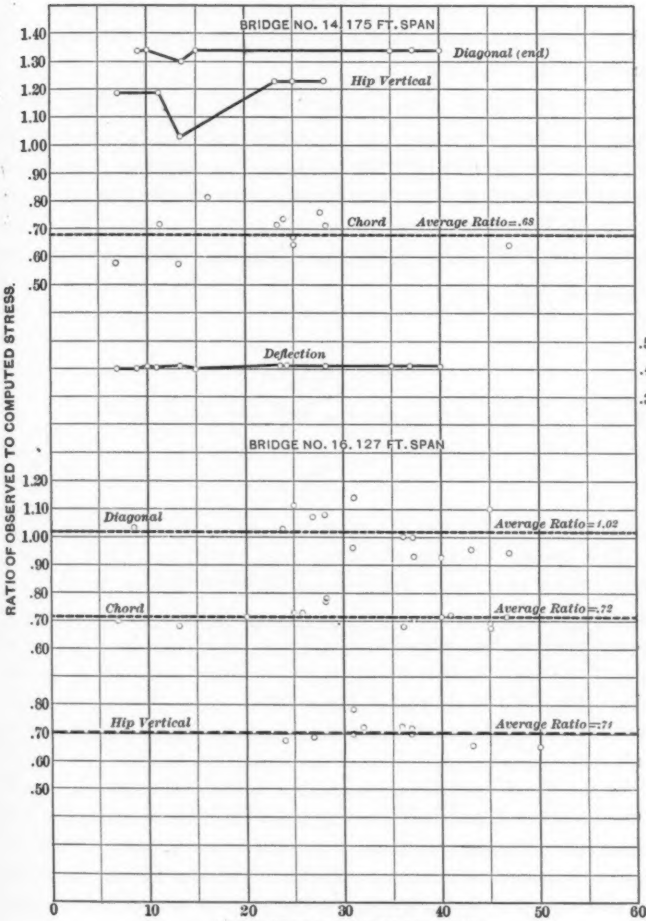

FIG. 13.

$r_, = \frac{a^2}{4\, y_o}$, whence, at this point, the centrifugal force is, approximately, $F = \frac{4\, M v^2\, y_o}{a^2}$. If $y_o = \frac{1}{2400}$ of span, and $v = 90$ ft. per second, or about 60 miles per hour, then $F = \frac{0.84}{a} P$, in which P = weight of moving body.

Thus, for a 25-ft. span, $F = \frac{0.84}{12.5} P = 7\%$ of weight; for a 50-ft. span, $F = 3.3\%$ of weight. As the actual deflection is somewhat increased by the centrifugal force, and hence the sharpness of curvature likewise increased, the effect is really somewhat greater than the above.

Dr. H. Zimmermann has treated this problem rigidly,* and a formula closely approximate to his exact formula (practically correct for values of $\frac{g\, a^2}{4\, v^2\, y_o} > 10$) is:

$$F = P \frac{1}{\frac{g\, a^2}{4\, v^2\, y_o} - 3},$$

or, with $y_o = \frac{1}{2400}$ of span, and $v = 90$ ft. per second, it becomes

$$F = P \frac{1}{\frac{a}{0.84} - 3},$$

a formula practically exact for values of a greater than about 8 ft.

This formula gives for the above cases, 8.7% and 3.7%, respectively. For a 15-ft. span, the increase would be 16%, with the same assumption as to the ratio of deflection to span length. For such short spans, however, the deflection would not, as a rule, be so great as $\frac{1}{2400}$ of the span, so that no higher value would be likely to be obtained than for a 25-ft. span. For a 100-ft. span, $F = 1.7$ per cent. The effect of a uniform load would be less than that of a concentrated load, as the average curvature is in that case much less.

By the above approximate methods it is shown that theory, as well as the results of experiments, indicates that the effect of speed alone is of no practical importance unless it be for very short spans. For such spans experimental data are not available. The best formula known to the author is that by Zimmerman. The effect of camber is to reduce the centrifugal force, but the small value of the force indicates that the benefit derived in this direction from cambering ordinary spans is small.

* "Die Schwingungen eines Trägers mit bewegter Last," by Dr. H. Zimmermann, Berlin, 1896.

COMPARISON OF MEAN STRESSES AND DEFLECTIONS FOR VARIOUS SPEEDS.
TRUSSES.

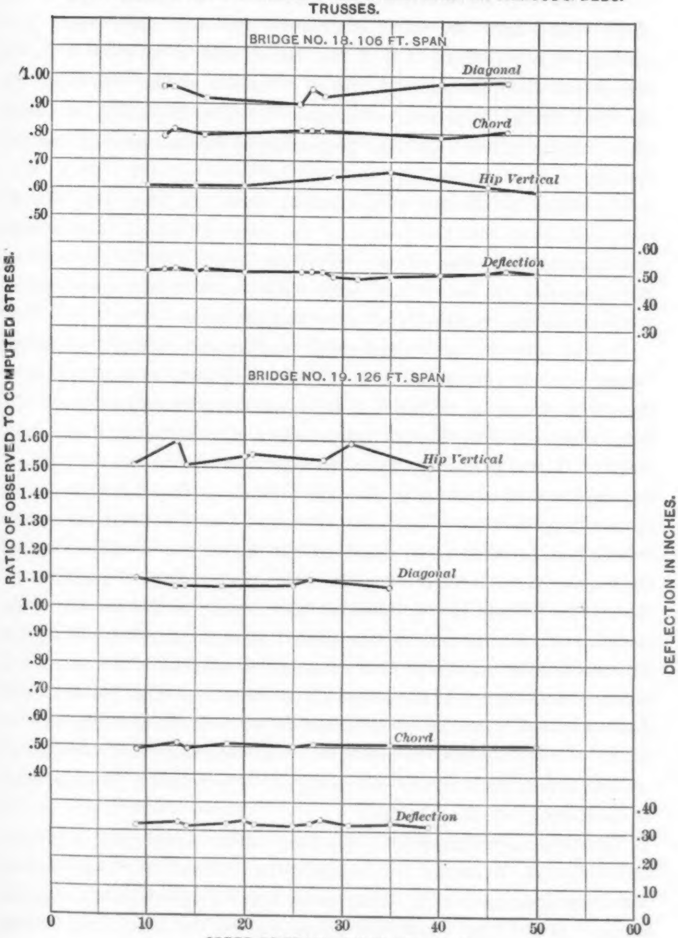

FIG. 14.

Vibrations in Girders and Trusses.

It will be convenient in discussing the subject of vibration, to treat the girders and trusses separately. As a rule the girders were of too short span-length to show cumulative vibration, the causes producing vibration acting but a few times, and usually at a much slower rate than the natural rate of vibration of the structure. In the truss bridges, however, conditions were often favorable for cumulative vibration, and this point is considered in that part of the discussion.

Vibrations in Girders.—The typical curves on page 417, already described, show that the feature of the tests which is of chief interest and importance is that of the vibrations caused by the locomotive. The results of all tests of any value have therefore been presented in Table No. 3, with particular reference to this point.

In the column headed deflection, and in the column containing values of stress per square inch, the several quantities were found by measuring the mean ordinates of the respective vibratory curves at the point where the deflection, or stress, was a maximum under the locomotive. The "percentage added by vibration" is the ratio of one-half the amplitude of vibration to the mean ordinate as thus determined. In the short spans the "vibrations per second" were not very accurately determined, as often there would be but one or two vibrations during the passage of the locomotive. The other columns are self explanatory. Where the records are of doubtful value, owing to the inertia of the instruments, a question mark is placed after the figures; but figures have not been given except where a careful inspection of the curves has satisfied the author that the actual vibrations were a large percentage of those recorded. Records of deflection are poor for many of the experiments on spans up to 45 ft. in length. The experiments on each bridge are arranged in the order of increasing speed of trains. The condition of track was good on all bridges.

The percentage of increase in the flange stress should agree in general with that in the deflection. An examination of Table No. 3 shows that, while the agreement is in many cases not very good for the shorter spans, it is good for the longer spans, and on the whole, considering the great difference between the two kinds of instruments used, is sufficient to show that the results are entirely reliable as to general indications. Several experiments have been omitted from the table,

the vibrations being too small to be of any consequence. All such tests, however, are accounted for in the column of remarks.

The principal points brought out by the data in Table No. 3 are:

(1) The increase in deflection, or flange stress, due to vibration is about the same in all spans from 25 to 55 ft. in length; and, neglecting two or three doubtful cases, has a maximum value of about 50% for speeds of 40 or 50 miles per hour. For spans of 60, 70 and 85 ft. in length, the maximum percentages are 22, 28 and 17 respectively.

(2) Speeds greater than about 20 miles per hour for short spans, and 35 miles per hour for long spans, almost invariably caused large vibrations.

(3) The rates of vibration of the bridges for high speeds and large amplitudes agree approximately with the rates of revolution of the locomotive drivers, and show quite clearly that the chief influence in producing vibration is the effect of the locomotive counterweights.

(4) There are undoubtedly other causes of vibration, such as roughness of track, flat wheels, etc. This is shown to some extent by the results from comparatively slow speeds, where the vibrations produced are much more rapid than the rate of rotation of the drivers. The presence of an iron floor system in the through girders, Nos. 10 and 12, appears to make no difference in the vibration. The panel length in both cases is about 14 ft., a very favorable length for vibration in Bridge No. 12, where a locomotive with 56-in. drivers was used.

(5) Different types of locomotives, and locomotives differently balanced, will undoubtedly produce different amounts of vibration, but in these tests there was little variety in locomotives or in counterbalancing. The ten-wheel engines appear to have caused more vibration in some cases than the American type, although the difference is hardly enough to warrant a general statement. The tests on Bridges Nos. 11 and 12 were all made with a ten-wheel engine with small drivers, but the resulting vibrations are rather low.

(6) As to a law of variation of vibration with speed, the experiments were not numerous enough to determine this point. In a general way vibration increases with speed, but apparently not as rapidly as the square of the speed, as the theory of centrifugal force would suggest. An attempt was made to discover any possible regularity in variation by plotting the experiments, but the accidental variations were too great to make the diagrams of any value.

(7) In the through girders, Nos. 10 and 12, considerable vibration was caused in some cases by loaded freight cars, although the panel length of 14 ft. is not favorable to such action (see Fig. 5).

Vibrations in Trusses.—Table No. 4 gives the principal data from the experiments on trusses, arranged as before, in the order of increasing speed for each bridge. As has already been explained, the extensometers were used on the lower chord (usually at the center), the diagonal nearest the center, and the hip vertical, the various members tested being indicated in the small diagrams in the table by heavy lines. Stresses, and percentages of increase due to vibration, are given for these members, together with corresponding values for the deflections. Where the vibrations were of any consequence, they agreed in rate for the deflection, chord stress, and stress in the diagonals, and also, usually, for the hip vertical.

In some cases the relative vibration was greatest before or after the maximum deflection occurred, and for some of these cases two values for vibration are given, one for the maximum vibration, and the other for vibration at time of maximum deflection.

The effect of vibration will be discussed first with reference to the deflection only, the relation between stress and deflection being taken up later. The salient points in the tests on each structure will be briefly noted.

Bridge No. 13 is one of the worst for vibrations. Under the locomotive there are many values above 15%, the highest being 20%, and highly cumulative (see Fig. 7). These large vibrations occur for speeds of 27 to 32 miles per hour, and are all for freight trains. The freight locomotives are much larger than the passenger, and are ten-wheelers, which facts may account for the smaller vibrations for speeds above 32; but it is more likely that this is due to the fact that the higher speeds do not accord so well with the natural rate of vibration for the loaded bridge. Fig. 6 is a record from a passenger train and shows much less vibration than Fig. 7, although the speed is much higher. Several curves from this bridge show large cumulative vibrations from loaded cars, but in no case is the total deflection equal to that produced by the locomotive. The conditions in this bridge are very favorable to vibration. The panel length is about equal to the circumference of a locomotive wheel, and, furthermore, the stringers are shallow and the structure is rather light and flexible. Passenger cars and empty freight cars cause no vibration.

Bridge No. 14 is a modern structure. The trains at this place were very light, as were also the locomotives. The maximum vibration was caused by a passenger engine at a speed of 47 miles per hour. Speeds below 15 miles per hour did not cause noticeable vibration.

Bridge No. 15 is a light structure. Speeds below 18 miles per hour gave little vibration. The maximum for locomotives is about 20% for a speed of 42, and that for cars is 28% at 28 miles per hour. In experiment No. 145, with a speed of 45, the vibration is greatest before the locomotive reaches mid-span, showing that this speed was probably too high for maximum cumulative effect. This bridge has a wooden floor, and a panel length of 17 ft. 4 ins., conditions very favorable for vibrations, both for cars and locomotives.

In the tests on Bridge No. 16, in a large number of cases, there was considerable vibration caused by the engine. Few records show much effect of cars, but there were very few freight trains observed here at speeds above 20 miles per hour. The floor system is not as flexible as in some of the other bridges in which wooden stringers are used, but the panel length is favorable for vibration, and this may partially account for the comparatively high percentage for so large a number of tests. The records from this structure show that speeds may vary within quite a large range, and still have conditions favorable for vibrations.

Bridge No. 17 is a thoroughly modern one with stiff lower chords and small deflection, but it shows fully as great vibrations from passenger engines as the lighter bridges. Freight trains were probably too slow here to cause much vibration.

Bridges Nos. 18 and 19 are also modern structures, No. 19 being very similar to No. 17. They were experimented upon by the special train before mentioned. The 106-ft. span shows 23% vibration under the locomotive for a speed of 25 miles per hour, and less for higher speeds; but the loaded cars produced much vibration at the high speeds. The panel length in this case is nearly equal to the driver circumference, and this may have influenced the vibration due to the engine. The vibration due to the cars cannot, however, be attributed to the stringers, for the rate of vibration implies a distance passed over of about 25 ft. for each vibration, while the panel length is 17 ft. 8 ins. It possibly may have been due to rough track near the bridge.

With Bridge No. 20 few experiments were made at sufficiently high speeds to develop vibrations, none at a speed of less than 26 miles per

hour causing appreciable disturbances, although ten such tests were made. The panel length of 25 ft. is quite unfavorable for vibration, the locomotive used having 56-in. drivers.

In the next four cases most of the records of the deflectometer were obliterated by dampness, and the records of the chord stresses must be used in lieu of anything better.

There is in the first three of these bridges slightly less than the usual amount of vibration, but the difference is not greater than would be accounted for by the lower speeds here observed. It was expected that No. 21 would have considerably less vibration than No. 22, owing to its much greater rigidity, but the tests failed to show much difference. In none of these three bridges was the panel length favorable to vibration. In the last case no observations were made on the chord, and but three of the records of the deflectometer are available. The results from the diagonal indicate rather high vibration, amounting to 26% in one case. The speeds are somewhat higher here than in the other three cases.

Cumulative Vibration.—In all cases where large vibrations were produced in the trusses tested, they were, without doubt, more or less cumulative, and highly so in the worst cases; as, for example, in the test illustrated in Fig. 7. The chief cause of vibration under the locomotive was undoubtedly the counterweights, the effect of which was increased in some cases by the deflection of the stringers; and the results show that cumulative vibration from this cause can occur for a comparatively wide range of speed, the rate of vibration increasing with the speed. True cumulative vibration occurs only when the impulses are timed to agree with the rate of vibration of the structure; but it is to be noted that as the locomotive passes upon a bridge the natural rate of vibration changes gradually from that of the empty bridge to that of a fully loaded one. The speed of the train may, therefore, vary between quite wide limits and yet cause cumulative vibration. At the same time, this change in rate of vibration of the bridge limits to a considerable extent the duration and intensity of the vibrations. It is quite noticeable, for example, in Bridge No. 13, that at the higher speeds the vibration was usually of the greatest relative intensity before the locomotive reached the center of the bridge, while with the slower speeds it was greatest with the bridge fully loaded (see Figs. 6 and 7). Other cases of high vibration occurring before the maximum deflection was reached are noted in the table (see also Fig. 8). In the case of vibrations produced by a uniformly loaded

train, the rate remains constant, and the vibrations often become highly cumulative. Speeds too low for cumulative effect will cause non-cumulative vibration, as in plate girders, which increases in intensity with decrease in span. Speeds too high for cumulative effect will cause shocks to stringers and other primary members, which will be absorbed before reaching the main truss.

Professor Robinson has thoroughly discussed the subject of cumulative vibration,* and in his paper derives an approximate formula for the time of vibration of loaded bridges. His formula for a uniformly distributed load is:

$$t = 2\,\pi\sqrt{\frac{2\ W\,d}{3\,g\,P}}$$

in which $t =$ time for one complete vibration; $W =$ total load, including weight of bridge; and $d =$ central deflection produced by any distributed load P. He has further shown the effect of car and locomotive springs in reducing the time of vibration of bridge and train, and presents a formula taking account of this effect. The author has made calculations by this formula, and also by a somewhat different one of his own, but with very unsatisfactory results, especially in the case of the locomotive. Calculations based on constants obtained from the railroad company, would call for a rate of vibration of the locomotive on its springs of about 2 per second, whereas the rate was found by actual observation to be between 4 and 5. It, however, can be easily shown experimentally that a combination of two elastic systems will vibrate as a single system at a rate somewhat slower than that of the most slowly vibrating component. Without attempting to take account of this effect of the springs of the locomotive, or cars, the author has calculated, by the above formula, the rate of vibration for each truss when loaded with the heaviest locomotive actually observed in each case, followed by a loaded freight train weighing 1 600 lbs. per lineal foot (about the actual weight). These values are given in the summary of tests on trusses.

Summary of Tests on Trusses.—In Table No. 5 is presented a summary of the principal data of the tests on trusses. Detailed information concerning the trusses is given in Table No. 2. In the third column of the table are given the ratios of deflection to span-length, under the average heavy train observed, to serve as a measure of the flexibility of

* *Transactions*, Am. Soc. C. E., Vol. xvi, p. 42.

the trusses under the actual traffic. As the locomotives varied considerably in weight, these figures do not show the relative flexibility of the bridges under like conditions.

The most noteworthy feature in these results is, perhaps, the comparative uniformity in the maximum percentages added to deflection by vibration. A reference to the details of the tests in Table No. 4, shows that the rate of vibration of the structures agrees quite closely with that of the revolution of the drivers, indicating that, as in plate girders, the chief cause of vibration is the unbalanced wheels. With the exception of Bridge No. 23, the only ones in which the vibration varies much from an average value are Nos. 14 and 19. These are among the stiffest in the list, but there are others having high vibration which are equally stiff, and it can hardly be concluded that stiffness is the cause of the small amount of vibration in these two cases. In fact, there is not enough variation in the results to enable conclusions to be drawn respecting the influence of any of the varying conditions.

These tests do indicate, however, that the vibration in almost any truss of the span-lengths here considered, is very likely to be as high as 20%, but that it is not likely to greatly exceed this percentage under ordinary traffic. Considering the large number of experiments and the variety of trusses, it would seem that the maximum might reasonably be placed at 25% for trusses of from 100 to 200 ft. in span-length, and perhaps for longer spans, as the tests do not indicate any tendency to an increase in vibration with increase in span.

The minimum speed causing vibration exceeding 10% of the static deflection, varies from about 20 to about 36 miles per hour, averaging, say 30. In two cases, Nos. 13 and 18, both flexible bridges, the maximum speed causing excessive vibration, was reached and passed. In all other cases the highest observed speed caused more than 10% vibration. The calculated rates of vibration are evidently too high. In Nos. 13 and 18 they correspond with the maximum velocity of train that causes excessive vibration, while in Nos. 22 and 23 they are less.

As regards vibration caused by cars, very few cases were noted for each bridge in which this was excessive, and, on account of the small number of loaded freight trains passing at high speeds, the records for Bridges Nos. 14, 17 and 20 are not of great value in this connection. It is noteworthy that the high percentages for cars are all for bridges in which the panel-length is very nearly equal to half a car length. Bridge

No. 22 is a very flexible structure, yet the vibrations produced by trains are invariably small.

Calculated Effect of Counterweights.—The practice of the Chicago, Milwaukee and St. Paul Railway in counterbalancing is to counterbalance for the revolving parts and for two-thirds of the weight of the reciprocating parts, the weight for the reciprocating parts being equally distributed among the drivers. Weights of the reciprocating parts have been obtained for two or three locomotives of each important class of engines involved in the tests on this company's bridges, and average values are herewith given, together with the amount of excess counterweight for each wheel, in terms of weight applied at crank-pin center. The class of locomotive is here indicated by its weight, which will enable it to be identified in the tables of tests. The centrifugal forces of these excess weights at speeds of 40 and 50 miles per hour have been computed, and are given in the following table in terms of percentage of weight on the driver. Account is taken of the fact that the counterweights are 90° apart and the resultant effect is here given. Nos. 5 and 6 are ten-wheel engines.

No.	Weight.	Diameter of drivers.	Stroke.	Weight on drivers.	Excess weight on each wheel.	Centrifugal Force (per cent).	
						40 miles per hour.	50 miles per hour.
	Lbs.	Ins.	Ins.	Lbs.	Lbs.		
1......	141 000	69	24	50 000	180	13	21
2......	154 000	62	24	54 000	152	13	21
3......	146 150	62	24	55 400	179	16	24
4......	178 200	68	24	64 100	203	12	19
5......	189 100	62	26	84 500	168	16	24
6......	170 600	56	26	76 700	155	19	30

For spans less than about 35 ft. in length the deflection caused by a locomotive is due almost entirely to the weight on the drivers at the time when it reaches its maximum value; and for such bridges these calculations would indicate an increase in deflection from the centrifugal force of the counterweights, amounting to from 20 to 30% for speeds of 50 miles per hour. For spans 50 ft. long the entire locomotive is on the bridge when the maximum deflection takes place, but the drivers still cause about two-thirds of this, and the calculable value of the effect of the counterweights would therefore be approximately 15% of the static deflection. For a 100-ft. span the values would be from one-third to

one-half of those given in the table, or from 7 to 15%, for velocities of 50 miles per hour. In all this computation, cumulative action has been neglected. Such action, undoubtedly, increases greatly the vibration in the longer spans.

The above-computed values are considerably less than the observed, but they suffice to show that, for short spans at least, vibration is certain to be large so long as locomotives are used which cannot be perfectly balanced. The excess of the observed vibration over the computed can, undoubtedly, be accounted for in many cases by the fact that the locomotive is vibrating on its springs as it passes upon the bridge. The author has endeavored on two occasions to learn, if possible, by observation from the locomotive, if such vibration took place. In both cases very considerable vibration was noted frequently as occurring at a rate of about 4 per second, and when it was most noticeable it agreed almost exactly with the rate of revolution of the drivers. Small bridges were crossed in some instances while this vibration was quite marked, and a test would, no doubt, have shown the effects of this.

Relation of Stresses to Deflections.

Chords.—An examination of the table will show that the percentage of increase in chord stress is usually about the same as in deflection. In a few cases there is considerable difference, sometimes the chord stress being the greater and sometimes the deflection.

Diagonals.—Here, again, in many cases the variation in stress agrees with that in the deflection, but on the average the increase in the former is considerably the higher. Considering only the effect of the locomotive, and discarding those tests in which the train moved in the wrong direction to cause a large stress in the diagonal, it will be found that the maximum percentage of increase for the diagonal is 30%, compared with 19% for the deflection in the same test. As the maximum stress in the center diagonal occurs before the maximum deflection, the increase in its stress due to vibration depends upon the vibration occurring slightly earlier than that which determines the increase in deflection as usually recorded (see Tests Nos. 33 and 27, on Bridge No. 13).

Hip Verticals.—The increase in stress in the hip vertical from vibration is in most cases much higher than in deflection, the maximum values varying between 15 and 47%, if one doubtful value of 72% is omitted. If most weight is given to the figures obtained from a large

number of experiments on the same bridge, an average maximum value would be about 40%, or approximately the same as found in plate girders of 40 or 50-ft. span lengths.

From the theoretical standpoint these results are about as would be expected. The action of a locomotive on a bridge when cumulative vibration takes place is two-fold: First, there is the local effect of each impulse from the drivers, and, second, the effect on all parts of the structure due to vibration. The effect on the chord members is mainly from the latter cause; that on diagonals from both, while the hip verticals and floor system near the end of the bridge would feel mainly the local effect. Stringers and floor beams near the center of the bridge would feel both the effect of the extra pressure due to vibration, and the impact of the drivers. No experiments were made on stringers or floor beams, but assuming them to receive the same impact as short girders, the total effect on these members would be something like 75 per cent. This may in reality be somewhat reduced by the more or less elastic supports of these girders.

Secondary Stresses and Computed Deflections.

As incidental to the main object of the tests, there were many cases where secondary stresses were determined by using both extensometers on the same member. As some of the results may be of interest, indicating as they do the possible magnitude of these stresses, they will be here given. In the plate girders, for example, the observed stresses varied from the computed as follows:

For 25-Ft. Span.

Top outside flange,	observed	value	= 130%	of computed.
Bottom	"	"	"	= 180%	"
" inside	"	"		= 45%	"

For 30-Ft. Span.

Top outside flange,	observed	value	= 125%	of computed.
Bottom	"	"	"	= 170%	"
" inside	"	"		= 70%	"

For 45-ft. spans and longer the flange stresses agreed very well with the computed, except in the through girders, where the lower flange stress was only about 70% of the computed value.

In general, the different bars of the same chord member had nearly the same stress, but all had more or less bending stress near the pin. This was not great for well-proportioned eye-bars, but was high for some very light bars on Bridge No. 15; and in the stiff chords of Bridge No. 17 the upper flange had, near the pin, only 75% as great a stress as the lower, while in a similar bridge, No. 19, the upper flange had less than 60% of the lower.

Diagonal bars pulled about equally, except in Bridges Nos. 13 and 16, where in one case the outside bar received 65%, and in the other only 50%, as much as the inside.

Hip verticals, where made of bars, were about equally stressed. In one case of stiff members the outside flange received 40% as much as the inside, and in another case 70 per cent.

In those bridges having an iron floor system the observed chord stresses were always from 20 to 30% less than the computed (see Figs. 2 and 3), whereas the observed diagonal stresses averaged about the same as the computed. The cause of this small chord stress was, undoubtedly, the action of the stringers as tension members. To find what effect this tension had on the floor beams, a few experiments were made on the end floor beam of one bridge. A stress as high as 3 000 lbs. per square inch was observed in the flanges, due to horizontal bending, this value being two-thirds as great as the stress due to the vertical load.

In the lighter bridges, and in the long diagonals of nearly all the bridges, considerable vibration occurred in the individual bars, causing high stresses in some cases, and being objectionable in general. This was not observed in the chord bars of the heavier structures.

It is doubtless true that secondary stresses are often much greater than stresses due to the vibration of the structure as a whole; and it is an open question whether more attention should not be paid to this feature. Experiments with the apparatus herein described can readily be made, and, if thoroughly carried out for a few well-designed trusses of any given form, would give a good idea of the amount of these secondary stresses. Theoretical investigations are for many cases quite practicable, but for others are too complicated for ready use, and actual tests are much more instructive. A large railway company could well afford to possess a few of the instruments such as described in this paper, as tests could easily be made which would be of much assistance in the development of standard designs.

Deflections have been computed for Bridges Nos. 18 and 19 by the formula involving the distortions of each member, $D = \Sigma \frac{p\,u\,l}{E}$. The results are, for No. 18, $D = 0.54$ in., and for No. 19, $D = 0.37$ in.; or, making approximate allowance for reduced chord stress on account of the action of the stringers, these values become 0.50 in. and 0.35 in., respectively. The observed values averaged about 0.49 in. and 0.33 in., respectively.

Results of Other Experiments.

The first published report of experiments on bridges under moving loads, that has come to the author's notice, is that of some tests made by Professor Fraenkel.* They were made with instruments similar to those described in this paper, but the extensometer as then constructed was not very reliable. The tests are of value chiefly in showing the amount of the secondary stresses in small riveted girders, but a few experiments were made which bring out the effects of high speeds. Deflection tests on four small girders showed an increase of from 2 to 48% in the deflection (including the effect of vibration) over that occurring at slow speeds. In a truss of 36.37 m. span the chord stress was not increased by an increase in speed; but there was considerable increase shown in web members and in the floor system.

The first really important series of tests are the well-known experiments made by Professor Robinson,† to which reference has already been made. The results of his tests on five bridges of from 141 to 156-ft. span-lengths show percentages of vibration for the locomotive varying from 19 to 28.6. One of these spans was a Whipple truss, while all others were Pratt trusses. All had panel-lengths approximately equal to the locomotive driver circumference; the stringers were in three cases wooden, and in the other cases iron, of a depth of from 24 to 26 ins. In two other bridges of 128-ft. spans, the vibration was 11.4% and 15.9 per cent. In these the rails rested on wooden floor-beams supported directly by the lower chord. Still another bridge, a very stiff, double-track span of 154 ft. 6 ins., with a panel length of 19 ft. 3¾ ins., was reported as having very little vibration. In all these tests the maximum vibration from cars was 50 per cent.

These experiments by Professor Robinson agree very well with those made by the author, except that for cars the maximum vibration

* *Der Civilingenieur*, 1881–1884 and 1887.

† *Transactions*, Am. Soc. C. E., Vol. xvi, p. 42.

observed by the latter was somewhat less. In no case, however, in any of the experiments, was the maximum deflection from a train as great as that from the locomotive.

A test of a riveted bridge of 35-m. span is reported by M. Cuenot.* The Manet apparatus was used, and it is stated that the stresses in the various members averaged about 22% higher under a speed of 25 km. per hour than at a low speed, or under static load. As the instrument is not one which gives a continuous record, it is to be supposed that this increase in stress includes all effects of vibration. The detailed figures show much irregularity in the percentages for the various members.

A series of experiments which will undoubtedly be of great value when completed is that now being carried on by M. Rabut on the Western Railways of France. M. Rabut reports his tests to show† that in a plate girder of 60-ft. span, with a static deflection of $\frac{3}{8}$ in., vibration will not increase the deflection by more than one-tenth. He also finds the same relative increase in stress as in deflection. Secondary stresses are found to be large. From some previous work done by him on eight bridges, of spans from 4 m. to 62 m. in length,‡ he comes to the following conclusions: At high speeds vibration is governed by wheel-base, cars, and panel-lengths; it increases with the speed. In large spans this vibration is only a small fraction of the static deflection. Excess of dynamic over static deflection is very small with main girders of long spans; it is sensible for beams and main girders for short spans, and considerable for stringers, especially near abutments. The time of vibration is independent of speed. Increase in speed scarcely affects mean deflection, but increases vibration, although this increase is, for large spans, only a small percentage of the deflection. As regards increase in mean deflection from increased speed, these results agree with the author's experiments, but the effects of vibration appear to be much less marked in the experiments of M. Rabut.

Professor M. A. Howe has described§ some observations of vertical and lateral deflection made on two bridges of 122 and 164-ft. span lengths. Very little vibration was observed in these experiments, the highest for trains being 7.5%, and for a single locomotive 10 per cent. The speeds were, however, all low, nearly all being below 25 miles per

* *Annales des Ponts et Chaussées*, July, 1891, p. 5.

† *Engineering*, August 13th, 1897, p. 183.

‡ *Le Génie Civil*, Vol. xxii, 1892, p. 88; abstracted in *Min. Proc. Inst. C. E.*, Vol. cxii, p. 391.

§ *Journal of the Association of Engineering Societies*, Vol. xiv, 1895, p. 513.

hour, which, from the author's tests, appears to be too low to develop much vibration.

A number of measurements of stresses in individual members were made on three bridges, in 1893, by Messrs. Hankenson and Ledger, using for the work the extensometer already mentioned.* Their experiments were mainly on hip verticals, but some records were obtained from other members. The curves show some vibration similar to that found by the author, and indicating excess of dynamic over static stress, but the increase appears to be considerably less. The diagrams from the lower chord show considerable vibration of the structure as a whole, but the record is mostly masked by the very rapid vibrations of the instrument, which were accompanied by a jarring sensation, as noted by the experimenters. Their instrument was evidently very sensitive to these rapid vibrations which, however, may have been really very small, as the author is inclined to believe was the case. The records from the diagonal indicated as much as 20% increase in stress due to vibration of the structure.

The latest mention of experiments on bridges is in the recent paper by Mr. Stone.† In this paper the author bases his coefficients for "immediate effect" entirely on results of tests on bridges in India. Average percentages from these results are given in the form of a curve on page 372. From this it appears that the experiments referred to showed an increase due to impact of from 50% for very short spans, to about 14% for spans in which the ratio of fixed to moving load is one-fourth, and about 7% where this ratio is one-half. While it is probable that these values for short spans are none too high and agree very well with the results of the author's experiments, the coefficients for the longer spans appear to be much too low for American practice, as indicated both by Professor Robinson's experiments, and those herein described. It would be very desirable to know more in detail of the nature of the tests referred to by Mr. Stone.

Conclusions.

The following is a summary of the principal results and conclusions which have already been mentioned:

(1) Speeds less than about 25 miles per hour are not likely to result in much vibration.

* *Engineering News*, May 9th, 1895, p. 300.

† "The Determination of the Safe Working Stress for Railway Bridges of Wrought Iron and Steel," by E. Herbert Stone, M. Am. Soc. C. E.; *Proceedings*, Am. Soc. C. E., May, 1898, p. 364.

(2) The increase in deflection due to vibrations, caused by locomotives running at speeds of 40 to 50 miles per hour, is likely to be 40 or 50% for girder spans of less than 50 ft. in length.

(3) This percentage decreases rapidly for longer spans, becoming about 25% as a maximum for 75-ft. spans.

(4) Owing to cumulative effect, the percentage is likely to be a maximum of 20 or 25, for spans from 75 ft. to 150 ft., or more, in length, but the experiments indicate no increase in percentage for increase in span.

(5) The relative increase in chord stress is about the same as in deflection; that in center diagonal is somewhat more than in the deflection; and in hip vertical it corresponds more nearly to that in girders of 40 to 50-ft. span-lengths.

(6) The effect of speed of application of the load on mean deflection was of no consequence in the spans tested (the increase in deflection from live load being due to vibration), although theory points to an appreciable increase from this cause in very short spans without camber.

(7) Secondary stresses are likely to be high in small girders with shelf-angles, and in some parts of trusses, and the discrepancy between observed and computed stress may be greater from this cause than from the dynamic effect of moving loads.

In conclusion, the author desires to acknowledge his great indebtedness to the various officials of the Chicago, Milwaukee and St. Paul Railway and of the Union Pacific Railroad, who kindly furnished facilities and information which have enabled these experiments to be carried out. Of those connected with the former road, special thanks are due to Onward Bates, M. Am. Soc. C. E., Engineer and Superintendent of Bridges and Buildings, for his great assistance and kindly interest throughout the entire work; to J. N. Barr, M. Am. Soc. C. E., Superintendent of Motive Power, and Mr. A. E. Manchester, Assistant Superintendent of Motive Power, for much information regarding weights of locomotives and cars; and to Division Superintendents Messrs. J. W. Stapelton and C. A. Cosgrave for special facilities in the way of trains. To George H. Pegram, M. Am. Soc. C. E., former Chief Engineer, and Mr. J. B. Berry, now Chief Engineer of the Union Pacific Railroad Company, the author is indebted for facilities and data in connection with the tests made on the four bridges of that road.

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TABLE No. 1.—DIMENSIONS OF GIRDER BRIDGES.

Reference No.	R. R. Co.'s No.	No. of spans.	Length of span, out to out.	Depth.	Distance of shelf-angles below upper flange.	Weight of one span, in pounds.	Date of construction.	Remarks.
1	Z 448½	1	25' 0"	1' 6½"	1' 0"	13 300	1892	Tested east span. Each span is composed of four girders.
2	L 98	3	25' 0"	1' 6⅝"	1' 0"	13 400	1894	
3	L 254	1	30' 0"	2' 6"	1' 3"	12 900	1894	
4	L 92	2	35' 0"	3' 0"	1' 2¼"	16 400	1894	Tested east span.
5	Z 450	2	35' 0"	3' 0"	1' 2¼"	16 600	1892	" "
6	Z 452	2	45' 0"	3' 6"	1' 2¼"	25 700	1892	" "
7	L 204	1	45' 0"	3' 6"	1' 2¼"	26 600	1895	
8	Z 318	1	55' 0"	4' 6"	2' 1½"	40 500	1895	
9	L 260	3	60' 0"	5' 3"	1' 2¼"	40 900	1894	Tested west span.
10	Z 484	3	70' 0"	6' 6"	Five panels; 24-in. beams; 19-in. stringers.	150 500	1890	Double track, with three girders. West span tested.
11	R 6	3	52' 0"	4' 4"	2' 2¼"	37 150	1896	Skew bridge. West span tested.
12	R 36	1	85' 0"	7' 0½"	Six panels; 30-in. beams; 22-in. stringers.	116 800	1896	

TABLE No. 2.—DIMENSIONS OF TRUSS BRIDGES.

Reference number.	Railroad company's number.	Number of spans.	Length of span, center to center.	Number of panels.	Panel length.	Depth.	Floor beams.	Stringers.	Hip verticals.	Weight of one span, in pounds.	Date of construction.	Remarks.
13	Z 312	5	158′ 3″	{ 8 { 1	17′ 9″ } 16′ 3″ }	25′ 0″	30-in.	{ Four 15-in. } { I-beams. }	Two 8-in. ⊔'s	212 500	1883	Skew bridge. Tested west span.
14	K 96	1	175′ 0″	8	21′ $10\frac{1}{2}$″	29′ 0″	$38\frac{1}{4}$-in.	27-in.	Two 6 x $\frac{11}{16}$-in.	285 400	1896	
15	L 206	2	104′ 0″	6	17′ 4″	23′ 0″	24-in.	Eight 3 x 16-in.	Two $2\frac{1}{2}$ x 1-in.	100 000±	1879	Trestle approach at west end. West span tested.
16	L 106	2	127′ 9″	7	18′ 3″	24′ 0″	27-in.	{ Four 15-in. } { I-beams. }	Two 6-in. ⊔'s	142 900	1886	East span tested. Trestle approach at west end.
17	L 90	1	123′ 6″	6	20′ 7″	27′ 0″	42-in.	Four 30-in.	Two 4 x $1\frac{1}{4}$-in.	179 500	1893	Lower chord stiff. Trestle approach at each end.
18	R 30	1	106′ 0″	6	17′ 8″	12′ 0″	Two 32-in.	22-in.	Two 10-in. ⊔'s	143 300	1896	Pony truss.
19	R 14	1	126′ 0″	6	21′ 0″	26′ 6″	$39\frac{1}{2}$-in.	$28\frac{1}{4}$-in.	{ Four 5 x $3\frac{1}{2}$-in. L's. } { One $8\frac{1}{2}$-in. pl }	176 000	1896	Lower chord stiff.
20	Columbus Bridge.	1	200′ 0″	7	28′ 7″	{ 24′ 0″ to 38′ 8″ }	42-in. (suspended)	45-in.	....	265 000±	1893	Pegram truss. East span of bridge.
21	Columbus Bridge.	1	147′ 6″	6	24′ 7″	{ 23′ 0″ to 30′ 0″ }	42-in. (suspended)	45-in.	....	165 000±	1893	Pegram truss. Second span from east end.
22	Elkhorn R. Bridge.	8	147′ $1\frac{1}{2}$″	11	13′ $4\frac{1}{2}$″	21′ 0″	20-in. (suspended)	....	Two 4 x $\frac{7}{8}$-in.	175 000±	Old spans	Tested east one of old spans — third span of bridge.
23	Elkhorn R. Bridge.	1	146′ $6\frac{3}{8}$″	9	16′ $3\frac{3}{8}$″	24′ 0″	30-in. (suspended)	I beams.	Two 3 x $1\frac{1}{8}$-in.	175 000±	....	Trestle approach at east end.

TABLE No. 3.—RESULTS OF TESTS ON PLATE GIRDERS.

Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	Deflection, in inches.	Percentage added to deflection by vibration.	Flange stress, in pounds per square inch.	Percentage added to flange strains by vibration.	† Vibrations per second.	Revolutions of drivers per second.	Remarks.
No. 1.—25-ft. span.	49	F. E.	824*	....	....	20	......	....	7 000	28	4	......	Vibrations irregular.
	46	F. E.	659*	189 100	62	36	0.12	....	4 000	25	4	3.4	
	45	P. W.	431	....	....	45	......	....	5 700	47	8	......	Excessive instrumental vibrations.
	52	P. W.	......	....	....	46	......	....	5 200	35	5	......	
	48	P. E.	311	154 000	62	50	......	....	4 300	25	5	4.6	
	47	P. E.	725	178 200	68	50	......	....	5 200	42	5	4.2	
No. 2.—25-ft. span.	234	F. W.	480	146 150	62	24	0.125	16	4 500	26	Irreg.	2.2	
	261	F. E.	358	141 000	69	25	0.10	20	3 800	47?	4	2.1	
	260	P. W.	538	141 000	69	27	0.11	15	4 800	35?	4	2.2	
	232	F. W.	496	146 150	62	30	0.12	25	4 700	30	3	2.8	
	237	F. W.	487	146 150	62	30	0.13	25	4 200	?	......	2.8	
	233	P. W.	546	141 000	69	35	0.11	35	4 500	?	......	2.9	
	238	F. E.	442	146 150	62	37	......	....	4 400	50?	......	3.4	
	235	P. E.	367	141 000	69	50	0.10	40	5 000	75?	5	4.1	
	236	P. W.	541	141 000	69	....	0.10	30	3 800	26	5	......	
No. 3.—30-ft. span.	162	F. E.	439	146 150	62	16	0.12	0	4 000	15	2	1.5	
	157	F. E.	442	146 150	62	23	......	....	3 700	10	2.5	2.2	
	160	F. W.	358	141 000	69	26	......	....	3 700	13	......	2.2	
	163	F. W.	480	146 150	62	33	......	....	4 000	30	3	3.0	
	159	P. W.	546	141 000	69	41	......	....	3 800	34?	4	3.4	
	164	P. E.	360	141 000	69	41	......	....	4 300	46	4	3.4	
	158	P. E.	367	141 000	69	43	......	....	3 800	35	4	3.6	
	161	P. W.	541	141 000	69	47	......	....	3 800	40	4	3.9	
No. 4.—35-ft. span.	223	F. W.	......	....	....	....	......	....	3 500	17	5	......	Ten other experiments with speeds from 4 to 24 miles per hour gave no increase in stress due to vibration greater than 10 per cent.
	220	F. W.	553	146 150	62	25	0.15	16	3 800	18	5	2.3	
	224	P. W.	545	141 000	69	27	......	....	3 300	30	4	2.2	
	213	P. W.	367	141 000	69	29	......	....	4 000	20	3	2.4	
	214	P. E.	365	141 000	69	35	0.13	20	4 000	30	3	2.9	
	211	F. E.	486	146 150	62	35	0.15	16	4 000	25	3	3.2	
	224½	P. E.	541	141 000	69	37	......	....	3 400	18	3.5	3.1	
	218	P. W.	365	141 000	69	41	0.15	25	3 000	50?	4	3.4	
	216	P. W.	362	141 000	69	43	0.14	21	3 800	40?	4	3.6	
No. 5.—35-ft. span.	62	F. E.	824*	....	....	9	0.20	0	5 800	0	......	......	
	42	F. W.	......	....	....	20	......	....	4 200	7	......	......	
	60	F. W.	715*	189 100	62	25	......	....	3 700	0	......	2.3	
	61	F. W.	639*	....	....	28	......	....	3 200	12	Irreg.	Irreg.	
	58	F. E.	648*	....	....	32	0.18	11	4 800	21	3.5	......	
	43	P. W.	311	154 000	62	33	......	....	4 200	60?	3.5	3.1	
	44	P. W.	722	178 200	68	33	......	....	4 000	26?	7	2.8	Instrumental vibration.
	55	F. E.	701*	189 100	62	39	0.17	45	4 500	49	4	3.6	
	53	P. W.	......	....	....	40	......	....	3 300	30	4	......	
	57	P. E.	340	....	....	50	......	....	4 000	40?	5	......	
	56	P. E.	732	178 200	68	60	......	....	4 200	95?	6	5.0	
No. 6.—45-ft. span.	63	F. W.	672*	189 100	62	19	0.23	6	4 200	10	4	1.7	
	"	" "	Cars.	....	....	....	0.10	15	1 600	15	3.3	......	
	66	F. W.	761*	189 100	62	20	......	....	4 400	9	......	1.8	
	"	" "	Cars.	....	....	....	......	....	2 400	15	3.5	......	
	70	F. W.	643*	....	....	21	......	....	4 100	15	Irreg.	......	
	72	F. E.	709*	189 100	62	27	0.29	34	5 700	33	3	2.5	
	71	F. E.	643*	....	....	29	......	....	4 500	31	3	......	
	74	F. E.	643*	....	....	29	0.27	13	5 000	12	3	......	
	75	F. E.	695*	189 100	62	33	0.24	42	5 000	65	3.3	3.1	
	64	P. W.	431	....	....	33	0.17	6	3 300	18	6	......	
	69	F. E.	739	178 200	68	38	......	....	4 500	44	4	3.2	
	65	P. E.	311	154 000	62	49	0.25	20	4 000	19	5	4.5	
	67	P. E.	725	178 200	68	50	0.29	24	4 700	42?	5	4.2	

* Ten-wheel engines; all others of American type.

† Roughly approximate when given in whole numbers.

TABLE No. 3—*(Continued)*.

Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	Deflection, in inches.	Percentage added to deflection by vibration.	Flange stress, in pounds per square inch.	Percentage added to flange strains by vibration.	† Vibrations per second.	Revolutions of drivers per second.	Remarks.
No. 7.—45-ft. span.	139	F. E.	83	....	....	23	0.15	0	2 600	0	......	......	
	142	F. W.	83	....	....	24	0.17	0	2 800	0	......	......	
	143	F. E.	504	146 150	62	24	0.20	0	2 700	9	5	2.2	
	141	F. W.	480	146 150	62	24	0.22	?	4 000	16	Irreg.	2.2	
	135	F. E.	442	146 150	62	27	0.19	19	3 300	15	3.0	2.5	
	138	F. W.	496	146 150	62	28	0.21	12	3 700	11	3.0	2.6	
	137	P. W.	546	141 000	69	35	......	....	3 000	20	3.7	2.9	
	134	P. E.	360	141 000	69	35	......	....	3 300	30	?	2.9	
	136	P. E.	538	141 000	69	37	......	....	4 000	19	3.5	3.1	
	144	P. E.	362	141 000	69	41	0.20	20	3 500	43	4	3.4	
	140	P. W.	541	141 000	69	59	......	....	3 200	50?	?	4.9	
No. 8.—55-ft. span.	2	F. W.	761*	189 100	62	37	......	....	4 700	15	3.7	3.4	
	3	F. W.	773*	189 100	62	37	......	....	4 750	21	3.7	3.4	
	5	F. W.	720*	189 100	62	44	0.30	12	4 000	20	4.6	4.1	
	6	F. W.	717*	189 100	62	44	0.31	20	4 500	22	4.3	4.1	
	4	F. E.	486	146 150	62	50	0.26	32	4 300	47	4.7	4.6	
	7	P. W.	731	178 200	68	54	0.26	25	3 900	31	5.0	4.5	
No. 9.—60-ft. span.	168	F. W.	496	146 150	62	24	0.22	11	3 800	16	3	2.2	Two other tests at 24 miles per hour gave 5% increase due to vibration.
	170	F. W.	442	146 150	62	30	0.25	20	4 100	18	3	2.8	
	172	F. W.	504	146 150	62	30	0.22	11	3 350	10	3.5	2.8	
	171	P. E.	361	141 000	69	32	0.22	10	3 200	16	3.2	2.7	
	166	P. E.	545	141 000	69	34	0.22	17	3 750	21	3.7	2.8	
	167	P. W.	360	141 000	69	44	0.22	12	3 800	16	4.3	3.6	
	169	P. W.	362	141 000	69	44	0.24	19	3 900	22	4	3.6	
No. 10.—70-ft. span.	83	F. E.	777*	189 100	62	35	......	....	2 200	18	4	3.3	Five other tests at speeds from 12 to 35 miles per hour gave no increase due to vibration greater than 8 per cent.
	81	P. E.	340	154 000	62	37	0.17	0	1 750	8	3.5	3.4	
	84	F. W.	781*	189 100	62	40	......	....	1 800	14	4.5	3.7	
	"	" "	Cars.	....	....	....	......	....	1 000	20	3	......	
	86	F. W.	767*	189 100	62	40	0.17	14	1 800	14	4	3.7	
	"	" "	Cars.	....	....	....	0.095	16	900	22	2.8	......	
	87	F. W.	715*	189 100	62	43	0.16	19	1 700	28	4	4.0	
	88	F. W.	668*	189 100	62	43	0.16	25	1 700	23	4	4.0	
	76	P. W.	......	....	....	47	0.12	17	1 200	21	4.5	......	
	80	P. E.	......	....	....	54	0.18	13	1 700	23	5	......	
No. 11.—52-ft. skew span.	316	E.†	454‡	170 600	56	12	0.27	0	4 000	5	......	1.2	
	315	W.	......	....	....	13	0.25	0	3 900	5	......	1.3	
	317	W.	......	....	....	20	0.27	0	4 000	7	......	2.0	
	319	W.	......	....	....	28	0.27	9	4 000	12	3.0	2.9	
	318	E.	......	....	....	29	0.27	?	4 100	12	3.0	3.0	
	320	E.	......	....	....	40	0.27	20	......	......	4.3	4.1	
	321	W.	......	....	....	43?	0.26	15	4 000	15	4.6	4.4	
	322	E.	......	....	....	43	0.27	18	4 000	21	4.6	4.4	
No. 12.—85-ft. span.	309	W.	......	....	....	20	0.35	3	2 700	3	4	2.0	Four other experiments at speeds from 6 to 17 miles per hour gave no appreciable vibration.
	"	"	Cars.	....	....	....	0.22	11	1 600	9	3	......	
	311	W.	......	....	....	23	0.35	6	2 700	Small	5	2.3	
	313	W.	......	....	....	31	0.35	9	2 750	6	3.5	3.2	
	310	E.	......	....	....	35	0.35	10	2 700	9	4.0	3.6	
	312	E.	......	....	....	41	0.35	17	2 700	15	4.4	4.2	
	314	E.	......	....	....	53	0.34	14	2 700	13	6	5.4	
	"	"	Cars.	....	....	....	0.22	14	1 600	16	......	......	

* Ten-wheel engines.

† All experiments on bridges Nos. 11 and 12 were made with a special train which always faced toward the east.

‡ Engine 454 is a ten-wheel engine.

TABLE No. 4.—Results of Tests on Trusses.

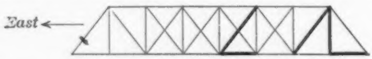
Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	Deflection. Mean deflection, in inches.	Deflection. Percentage added by vibration.	Stress in Chord. Mean stress, in pounds per square inch.	Stress in Chord. Percentage added by vibration.	Stress in Diagonal. Mean stress, in pounds per square inch.	Stress in Diagonal. Percentage added by vibration.	Stress in Hip. Mean stress, in pounds per square inch.	Stress in Hip. Percentage added by vibration.	Vibrations per second.	Revolutions of drivers per second.
No. 13.—158-ft. span.	8	F. W.	714	189 100	62	27	....	....	4 200	24	......	....	......	....	2.5	2.5
	26	F. W.	777	189 100	62	27	0.68	13	......	....	6 700	13	......	....	2.7	2.5
	“	“ “	Cars.	....	....	....	0.42	20	......	....	3 300	27	......	....	2.4	....
	28	F. W.	747	189 100	62	27	0.65	17	......	....	6 300	25	......	....	2.4	2.5
	“	“ “	Cars.	....	....	....	0.44	26	......	....	3 500	40	......	....	2.2	....
	37†	F. W.	709	189 100	62	27	0.70	20	......	....	6 800	21	......	....	2.5	2.5
	10	F. E.	773	189 100	62	28	0.75	19	4 300	19	......	....	......	....	2.8	2.6
	“	“ “	Cars.	....	....	....	0.58	26	2 800	25	......	....	......	....	2.5	....
	14	F. E.	720	189 100	62	28	0.72	14	3 700	11	......	....	......	....	2.8	2.6
	“	“ “	Cars.	....	...	....	0.50	30	2 700	33	......	....	......	....	2.8	....
	23	F. W.	788	189 100	62	28	0.71	11	4 000	10	......	....	......	....	3.2	2.6
	31	F. E.	......	....	....	28	0.71	17	......	....	......	....	......	....	2.4	....
	“	“ “	Cars.	....	....	....	0.57	23	......	....	......	....	......	....	2.0	....
	12	F. W.	793	189 100	62	29	0.67	6	3 700	8	......	....	......	....	2.8	2.7
	25	F. W.	720	189 100	62	30	....	....	3 800	13	......	....	......	....	....	2.8
	38†	F. E.	747	189 100	62	30	0.69	12	......	....	2 000	55	......	....	3.0	2.8
	11	F. E.	788	189 100	62	31	0.80	6	......	....	......	....	......	....	....	2.9
	“	“ “	Cars.	....	....	....	0.45	18	......	....	......	....	......	....	....	....
	32	F. E.	788	189 100	62	31	0.72	10	......	....	5 200	20	......	....	3	2.9
	33*	F. W.	781	189 100	62	31	0.52 0.67	17 5	....	....	6 100	16	......	....	3	2.9
	27*	F. E.	794	189 100	62	32	0.57 0.73	14 10	....	....	5 400	15	......	....	3	3.0
	29	P. E.	721	178 200	68	33	0.64	9	......	....	5 000	10	......	....	3.3	2.8
	35	F. E.	720	189 100	62	33	0.71	5	......	....	5 700	11	......	....	3	3.1
	“	“ “	Cars.	....	....	....	0.51	16	......	....	4 300	21	......	....	2.5	....
	9	P. E.	734	178 200	68	37	0.80	5	4 400	7	......	....	......	....	3.7	3.1
	34	P. W.	731	178 200	68	37	0.66	8	......	....	4 400	14	......	....	4	3.1
	40†	P. E.	......	....	....	37	....	....	......	....	5 300	13	......	....	3.3	....
	13	P. W.	731	178 200	68	40	0.67	4	4 500	5	......	....	......	....	3.5	3.4
	22	F. W.	761	189 100	62	40	0.70	11	4 000	10	......	....	......	....	3.5	3.7
	24	P. W.	732	178 200	68	40	0.69	5	4 100	10	......	....	......	....	3.4	3.4
	39†	P. E.	721	178 200	68	40	0.67	5	......	....	4 100	22	......	....	4	3.4
	36	P. W.	......	....	....	47	0.68	5	......	...	6 200	11	......	....	4.5	....
	41	P. W.	722	178 200	68	50	....	....	......	....	5 800	24	3 200	72?	5	4.2

All freight engines are ten-wheelers.

* Maximum vibration occurs before maximum deflection in Nos. 33 and 27.

† Tests Nos. 37, 38, 40 and 39 were made on center diagonal, all others on end diagonal.

TABLE No. 4—(*Continued*).



Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	DEFLECTION.		STRESS IN CHORD.		STRESS IN DIAGONAL		STRESS IN HIP.		Vibrations per second.	Revolutions of drivers per second.
							Mean deflection, in inches.	Percentage added by vibration.	Mean stress, in pounds per square inch.	Percentage added by vibration.	Mean stress, in pounds per square inch.	Percentage added by vibration.	Mean stress, in pounds per square inch.	Percentage added by vibration.		
No, 14.—175-ft. span.	93	F. E.	......	....	....	16	0.41	5	2 500	4	......	....	......	....	2.6	....
	110	F. E.	426	126 600	57	20	0.46	2	......	....	3 000	7	......	....	3.3	....
	97	F. W.	403	121 000	57	23	0.41	0	2 200	0	......	....	2 800	5	....	....
	92	L. W.	637*	182 000	56	25	0.39	5	2 400	6	......	....	......	....	2.5	....
	94	P. E.	411	119 150	56	25	0.37	3	2 250	4	......	....	......	....	3	....
	99	F. W.	403	121 000	57	25	0.41	2	2 300	0	......	....	2 800	2	3.5	....
	101	F. W.	403	121 000	57	28	0.41	6	2 200	4	......	....	2 800	9	3.0	....
	90	F. W.	403	121 000	57	31	0.36	5	2 200	4	......	....	......	....	....	....
	95†	P. W.	407	121 000	56	31	0.35	3	1 800	0	......	....	......	....	....	....
	105‡	F. W.	403	121 000	57	34	0.41	2	......	..	3 400	4.	end..	....	3	....
											2 500	14.	c'nt'r	....	7	....
	103‡	F. W.	403	121 000	57	36	0.41	10	......	..	3 400	5.	end..	....	3	....
											2 500	16.	c'nt'r	....	7	....
	107‡	F. W.	403	121 000	57	40	0.41	5	......	..	3 400	3.	end..	....	....	....
											2 000	0.	c'nt'r	....	....	....
	96	P. E.	411	119 150	56	44	0.33	7	2 500	16	......	....	2 500	16	4	....
	91	P. W.	411	119 150	56	47	0.33	12	1 800	8	......	....	......	....	4.4	....

† Test No. 95 was made on end chord, all others on chord near center.

‡ Vibrations recorded for center diagonal were due to vibration of bar and not of bridge.

Seven unrecorded tests at speeds from 8 to 15 miles per hour showed no appreciable vibrations.

Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	Mean deflection, in inches.	Percentage added by vibration.	Mean stress in chord, in pounds per square inch.	Percentage added by vibration.	Mean stress in diagonal, in pounds per square inch.	Percentage added by vibration.	Mean stress in hip, in pounds per square inch.	Percentage added by vibration.	Vibrations per second.	Revolutions of drivers per second.
No. 15.—104-ft. span.	125	F. E.	480	146 150	62	23	0.41	0	5 700	5	......	....	......	....	....	2.1
	"	" "	Cars.	....	....	....	0.16	12	1 300	11	......	....	......	....	4.5	....
	130	F. W.	442	146 150	62	24	0.37	5	......	....	3 300	18	......	....	....	2.2
	118	F. W.	480	146 150	62	25	0.36	7	1 800	9	......	....	......	....	2.5	2.3
	114	F. E.	442	146 150	62	26	0.36	5	5 800	9	......	....	......	....	....	2.4
	"	" "	Cars.	....	....	....	0.26	10	3 000	18	......	....	......	....	....	....
	146	F. E.	....	....	....	27	0.39	6	......	....	5 500	5	6 800	9	3	....
	132	F. W.	439	146 150	62	28	0.38	13	6 500	17	2 100	38	......	....	3	2.6
	"	" "	Cars.	....	....	....	0.30	28	5 000	24	2 100	60	......	....	3	....
	148	F. W.	553	146 150	62	28	0.35	5	......	....	5 200	8	8 400	19?	3	2.6
	154	F. E.	487	146 150	62	29	0.35	6	......	....	......	....	......	....	3	2.7
	126	F. W.	504	146 150	62	30	0.39	12	4 300	12	......	....	......	....	3.1	2.8
	152	F. W.	442	146 150	62	31	0.36	13	......	....	......	....	8 500	16?	3.6	2.9
	149	F. E.	358	141 000	69	33	0.38	9	......	....	......	....	7 500	12	3	2.7
	"	" "	Cars.	....	....	....	0.23	17	......	....	......	....	......	....	2.5	....
	155	P. E.	546	141 000	69	33	0.37	12	......	....	......	....	5 000	20	3	2.7
	133	P. E.	546	141 000	69	34	....	....	......	....	8 500	8	......	....	4	2.8
	150	P. W.	365	141 000	69	37	0.35	14	......	....	......	....	......	....	....	3.1
	127	P. W.	362	141 000	69	37	0.37	10	4 300	12	......	....	......	....	4	3.1
	121	F. E.	496	146 150	62	39	....	....	4 400	23	......	....	......	....	....	3.6
	"	" "	Cars.	....	....	....	....	....	3 000	20	......	....	......	....	....	....
	115	P. W.	541	141 000	69	39	0.37	6	......	....	......	....	......	....	....	3.2

Six other tests at speeds from 7 to 18 miles per hour gave no appreciable vibration.

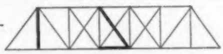
* Ten-wheel engine.

TABLE No. 4—(*Continued*).

Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	DEFLECTION.		STRESS IN CHORD.		STRESS IN DIAGONAL.		STRESS IN HIP.		Vibrations per second.	Revolutions of drivers per second.
							Mean deflection, in inches.	Percentage added by vibration.	Mean stress, in pounds per square inch.	Percentage added by vibration.	Mean stress, in pounds per square inch.	Percentage added by vibration.	Mean stress, in pounds per square inch.	Percentage added by vibration.		
No. 15. continued.	147*	P. W.	362	141 000	69	41	0.37	13	......	....	4 500	33	7 000	45	....	3.4
	113	P. E.	538	141 000	69	42	0.36	14	4 800	13	......	....	......	....	4	3 5
	120	P. E.	362	141 000	69	42	0.36	20	7 000	19	......	....	......	....	3.8	3.5
	122	F.W.	10	....	....	43	....	....	3 750	8	......	....	......	....	3.5	....
	123	F. E.	10	....	....	43	0.29	7	3 800	5	......	....	......	....	3.6	....
	117	P. W.	367	141 000	69	44	0.35	10	......	....	......	....	......	....	....	3.6
	129	P. W.	365	141 000	69	44	0.39	7	......	....	4 800	12	......	....	4	3.6
	145	P. E.	361	141 000	69	45	{ 0.27 0.38	19 10	......	....	4 800	30	6 000	30?	4	3.7
	124	P. E.	361	141 000	69	47	0.29	12	4 000	12	......	....	......	....	4	3.9

* Increase in stress in diagonal and hip due chiefly to vibration of the individual bars.

East ←

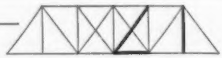


Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	Mean deflection, in inches.	Percentage added by vibration.	Chord: Mean stress	Percentage added by vibration.	Diagonal: Mean stress	Percentage added by vibration.	Hip: Mean stress	Percentage added by vibration.	Vibrations per second.	Revolutions of drivers per second.
No. 16—127-ft. span.	178	F. E.	442	146 150	62	20	0.42	7	2 800	7	......	....	......	....	4	1.8
	202	F. E.	492	146 150	62	24	0.43	6	......	....	4 800	6	2 300	15	3.3	2.2
	188	F. W.	584	146 150	62	25	0.35	8	3 200	6	......	....	......	....	2.7	2.3
	181	P. W.	360	141 000	69	28	0.36	7	2 900	10?	......	....	......	....	3.6	2.3
	184	F. E.	486	146 150	62	28	0.37	6	2 800	5	4 800	6	......	....	2.9	2.6
	"	" "	Cars.	....	....	....	0.14	21	800	25	500	80	......	....	4.7	....
	187	F. W.	439	146 150	62	28	0.34	12	3 000	5	......	....	......	....	3	2.6
	201	P. E.	367	141 000	69	31	0.38	21	......	....	4 600	22	2 500	40?	3.2	2.6
	195	F. W.	358	141 000	69	31	0.36	11	......	....	......	....	4 000	12	3.2	2.6
	190	F. E.	496	146 150	62	31	0·42	18	......	....	5 200	21	......	....	3.2	2.9
	193	F. E.	504	146 150	62	32	0.42	15	......	....	......	....	3 500	20	3.1	3.0
	179	P. E.	365	141 000	69	36	0.40	12	2 800	11	......	....	......	....	....	3.0
	200	F. E.	439	146 150	62	36	0.44	11	......	....	4 800	25	2 500	28	3.2	3.3
	"	" "	Cars.	....	....	....	0.31	15	......	....	1 900	58	......	....	2.7	....
	196	F. E.	480	146 150	62	37	0.43	4	......	....	......	....	3 500	14	3.4	3.4
	197	P. E.	541	141 000	69	37	0.41	7	......	....	4 600	15	2 300	11	3.7	3.1
	185	P. E.	361	141 000	69	37	0.39	15	2 900	14	4 800	23	......	....	3.5	3.1
	191	P. E.	546	141 000	69	40	0.37	11	......	....	4 300	23	......	....	3.7	3.3
	177	F. W.	496	146 150	62	40	0.39	11	2 900	17	......	....	......	....	4	3.7
	174	P. E.	362	141 000	69	41	0.40	18	3 000	16	......	....	......	....	3.7	3.4
	199	P. E.	538	141 000	69	43	0.38	13	......	....	4 600	19	2 100	19	4.1	3.6
	176	P. W.	367	141 000	69	45	0.37	11	2 500	12	......	....	......	....	4	3.7
	186	P. W.	362	141 000	69	45	0.37	17	2 800	16	3 900	28	......	....	3.8	3.7
	182	P. E.	545	141 000	69	47	0.36	20	2 900	17	4 500	18	......	....	4	3.9
	194	P. W.	545	141 000	69	50	0.36	14	......	....	......	....	3 000	35	4.2	4.1

Eight other tests at speeds from 8 to 26 miles per hour showed no appreciable vibration.

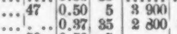
TABLE No. 4—(*Continued*).

Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	DEFLECTION. Mean deflection, in inches.	DEFLECTION. Percentage added by vibration.	STRESS IN CHORD. Mean stress, in pounds per square inch.	STRESS IN CHORD. Percentage added by vibration.	STRESS IN DIAGONAL. Mean stress, in pounds per square inch.	STRESS IN DIAGONAL. Percentage added by vibration.	STRESS IN HIP. Mean stress, in pounds per square inch.	STRESS IN HIP. Percentage added by vibration.	Vibrations per second.	Revolutions of drivers per second.



Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	Mean deflection, in inches.	Percentage added by vibration.	Mean stress (chord), lb. per sq. in.	Percentage added by vibration.	Mean stress (diagonal), lb. per sq. in.	Percentage added by vibration.	Mean stress (hip), lb. per sq. in.	Percentage added by vibration.	Vibrations per second.	Revolutions of drivers per second.
No. 17.—123-ft. span.	259	P. E.	538	141 000	69	27	0.28	5	......	....	......	....	2 400	17	4	2.2
	228	P. E.	546	141 000	69	27	....	....	......	....	......	....	2 750	7	4	2.2
	226	F. W.	353	....	....	29	....	....	......	....	......	....	3 400	26	....	....
	241	P. E	362	141 000	69	30	....	....	2 700	8	......	...	......	....	3	2.5
	257	F. E.	504	146 150	62	31	0.28	12	......	....	......	....	2 700	11	2.9	2.9
	251	F. E.	553	146 150	62	32	0.27	6	......	....	1 300	0	......	....	3.8	3.0
	"	" "	Cars.	....	....	....	0.20	12	2 700	11	......	....	......	....	3.6	....
	229	F.	487	146 150	62	....	....	....	......	....	......	....	3 000	26?	....	....
	240	P. W.	538	141 000	69	33	....	....	2 000	15	......	....	......	....	4	2.7
	252	P. W.	361	141 000	69	33	0.26	19	......	....	2 800	12	......	....	4	2.7
	250	P. W.	365	141 000	69	38	0.27	13	......	....	2 000	20	......	....	3.5	3.1
	246	P. W.	362	141 000	69	39	0.30	20	2 500	16	2 800	21	......	....	3.6	3.2
	254	P. W.	546	141 000	69	40	0.25	16	......	....	2 900	14	2 600	27	3.6	3.4
	243	P. W.	360	141 000	69	43	....	....	2 700	15	......	....	......	....	3.8	3.5
	225	P. W.	365	141 000	69	45	....	....	......	....	......	....	2 800	36	4.5	3.7
	230	P. W.	545	141 000	69	47±	....	....	......	....	......	....	2 600	31	5	3.9
	231	P. E.	......	....	....	47±	....	....	......	....	......	....	2 700	26	....	....

Eight other tests at speeds from 16 to 25 miles per hour showed very small vibration.



Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	Mean deflection, in inches.	Percentage added by vibration.	Mean stress (chord), lb. per sq. in.	Percentage added by vibration.	Mean stress (diagonal), lb. per sq. in.	Percentage added by vibration.	Mean stress (hip), lb. per sq. in.	Percentage added by vibration.	Vibrations per second.	Revolutions of drivers per second.
No. 18—106-ft. span.	299	W.*	454*	170 600	56	20	0.50	8	......	....	......	....	1 250	0	3.8	2.0
	292	E.	......	....	....	26	0.50	7	3 900	8	3 200	0	......	....	3.0	2.6
	295	W.	......	....	....	27	0.50	7	3 900	10	3 400	12	......	....	3.5	2.7
	293	W.	......	....	....	28	0.50	7	3 900	5	3 300	0	......	....	3.4	2.8
	301	W.	......	....	....	29	0.48	4	......	....	......	....	......	....	3.6	3.0
	303	W.	......	....	....	32	0.47	12	......	....	......	....	......	....	3.7	3.3
	300	E.	......	....	....	35	0.48	23	......	....	......	....	1 300	15	3.7	3.6
	294	E.	......	....	....	40	0.49	15	3 800	11	3 400	5	......	....	4	4.1
	302	E.	......	....	....	45	0.50	5	......	....	......	....	......	....	4.4	4.6
	"	"	Cars	....	....	....	0.36	35	......	....	......	....	......	....	3.0	....
	296	E.	......	....	....	47	0.50	5	3 900	5	3 400	9	......	....	3.6	4.8
	"	"	Cars.	....	....	....	0.37	35	2 800	32	1 200	46	......	....	2.7	....
	304	E.	......	....	....	50	0.50	5	......	....	......	....	......	....	....	5.1
	"	"	Cars.	....	....	....	0.36	35	......	....	......	....	600	42	3.1	....

Five other tests at speeds from 10 to 16 miles per hour gave no appreciable vibration.

Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	Mean deflection, in inches.	Percentage added by vibration.	Mean stress (chord), lb. per sq. in.	Percentage added by vibration.	Mean stress (diagonal), lb. per sq. in.	Percentage added by vibration.	Mean stress (hip), lb. per sq. in.	Percentage added by vibration.	Vibrations per second.	Revolutions of drivers per second.
No. 19.—126-ft. span.	276	E.	......	....	....	27	0.33	3	1 800	0	3 600	6	......	....	3.5	2.8
	284	E.	......	....	....	28	0.34	6	......	....	......	....	3 850	17	3.7	2.9
	286	E.	......	....	....	31	0.32	12	......	....	......	....	4 000	15	3.7	3.2
	278	E.	......	....	....	35	0.33	10	1 800	11	3 500	9	......	....	3.9	3.6
	288	E.	......	....	....	39	0.32	7	......	....	......	....	3 800	26	4	4.0
	"	"	Cars.	....	....	....	0.25	20	......	....	......	....	......	....	....	....
	280	E.	......	....	....	50	0.33	13	......	....	......	....	......	....	....	5.1

Ten other tests at speeds from 10 to 26 miles per hour gave no appreciable vibration.

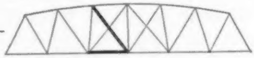
* All experiments on bridges Nos. 18 and 19 were made with a special train which always faced toward the east. Engine No. 454 is a ten-wheel engine.

TABLE No. 4—(*Continued*).

Bridge No.	No. of experiment.	Kind and direction of train.	No. of engine.	Weight of engine and tender, in pounds.	Diameter of drivers, in inches.	Speed of train, in miles per hour.	DEFLECTION. Mean deflection, in inches.	DEFLECTION. Percentage added by vibration.	STRESS IN CHORD. Mean stress, in pounds per square inch.	STRESS IN CHORD. Percentage added by vibration.	STRESS IN DIAGONAL. Mean stress, in pounds per square inch.	STRESS IN DIAGONAL. Percentage added by vibration.	STRESS IN HIP. Mean stress, in pounds per square inch.	STRESS IN HIP. Percentage added by vibration.	Vibrations per second.	Revolutions of drivers per second.
Seven-panel Pegram truss. Center chord member tested.																
No. 20—200-ft. span.	325	F. E.	774	169 500	62	20	0.41	6	2 200	9	......	....	......	....	....	1.9
	323	F. E.	799	169 500	62	....	....	....	2 900	14	......	....	......	....	3	....
	324	P. W.	......	....	....	24	0.57	5	2 750	18	......	....	......	....	2.5	....
	332	P. W.	890	226 800	69	28	....	....	3 200	11	......	....	......	....	2.5	2.3
	326	P. E.	1 800*	216 500	69	30	....	....	1 400	14	......	....	......	....	2.7	2.5
	331	P. E.	1 801*	216 500	69	44	0.32	12	3 300	10	......	....	......	....	2.5	3.6

* Engines 1 800 and 1 801 are ten-wheelers.

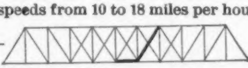
East ←



No. 21.—147-ft. span.	334	F. W.	779	169 500	62	11	....	....	2 750	5	1 600	10	......	....	2.4	1.0
	343	F. W.	771	169 500	62	22	....	....	3 000	5	4 000	11	......	....	2.7	2.0
	339	F. W.	773	169 500	62	23	....	....	2 400	10	3 800	13	......	....	2.7	2.1
	345	F. E.	799	169 500	62	25	0.51	10	4 000	10	4 100	12	......	....	2.7	2.3
	335	P. W.	845	213 000	69	25	....	....	2 800	7	4 800	6	......	....	2.5	2.1
	338	F. E.	670	169 500	62	25	....	....	2 800	7	4 200	12	......	....	2.5	2.3
	347	F. W.	700	169 500	62	25	....	....	2 800	7	3 300	6	......	....	2.5	2.3
	348	F. W.	722	169 500	62	27	....	....	2 800	14	3 300	12	......	....	3	2.5
	340	P. W.	771	169 500	62	30	....	....	3 000	0	3 600	11	......	....	2.6	2.8
	341	P. W.	1 801	216 500	69	32	....	....	3 500	17	4 000	12	......	....	2.8	2.6
	337	P. E.	890	226 800	69	40	....	....	2 600	12	4 900	10	......	....	3	3.3

Six other tests at speeds from 10 to 18 miles per hour gave no vibration.

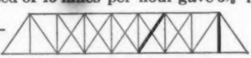
East ←



No. 22.—147-ft. span.	362	F. E.	641	169 500	62	20	0.85	16	5 800	17	2 500	18	......	....	2.1	1.9
	355	F. E.	792	169 500	62	21	0.81	16	5 800	15	2 500	18	......	....	2.6	2.0
	365	F. E.	700	169 500	62	23	....	....	5 800	10	2 800	9	......	....	2.3	2.1
	366	F. W.	773	169 500	62	24	....	....	5 800	4	3 700	5	......	....	2.2	2.2
	369	F. W.	799	169 500	62	25	....	....	5 700	15	3 800	8	......	....	2.4	2.3
	357	P. E.	1 801	216 500	69	27	....	....	6 800	7	3 300	12	......	....	2.2	2.2
	364	F. E.	722	169 500	62	27	....	....	5 200	11	2 800	4	......	....	2.4	2.5
	368	F. E.	......	....	....	27	....	....	4 500	18	2 100	17	......	....	2.2	....
	367	F. E.	643	169 500	62	31	....	....	6 500	3	2 800	0	......	....	....	2.9
	356	P. E.	777	169 500	62	40	....	....	6 100	18	2 800	?	......	....	3.0	3.7

Two other tests at a speed of 15 miles per hour gave 5% increase due to vibration.

East ←



No. 23.—146-ft. span.	375	F. W.	722	169 500	62	23	0.57	11	......	....	3 300	24	......	....	2.8	2.1
	380	F. E.	671	169 500	62	25	....	....	......	....	3 500	14	3 300	0	....	2.3
	372	P. E.	845	213 000	69	35	0.67	7	......	....	4 200	7	......	....	....	2.9
	379	F. W.	672	169 500	62	37	....	....	......	....	4 200	26	3 200	19	4	3.4
	376	P. E.	490	136 000	68	40	....	....	......	....	4 200	14	3 300	33	3.6	3.4
	371	P. W.	844	213 000	69	46	0.63	10	......	....	4 200	5	......	....	....	3.8
	378	P. W.	1 800	216 500	69	50	....	....	......	....	5 800	8	5 000	10	....	4.1
	381	P. W.	......	....	....	50	....	....	......	....	......	....	5 200	20†	....	....
	377	P. W.	775	169 500	62	52	....	....	......	....	4 800	25	3 600	47	4	4.8

Two other tests at speeds of 14 and 18 miles per hour gave no appreciable vibration.

†Due largely to vibration of bar.

TABLE No. 5.—SUMMARY OF TESTS ON TRUSSES.

Bridge No.	Span length.	Deflection under heavy train.	Ratio of deflection to span × 100 000.	Panel lengths.	Speeds causing over 10 per cent. vibration from the locomotive. Miles per hour.	Maximum percentage of increase in deflection due to vibration from the locomotive.	Rate of driver-revolution for speeds causing over 10 per cent. vibration. Number per second.	Rate of bridge vibration when vibration is over 10 per cent. Number per second.	Calculated rate of vibration of loaded bridge. Number per second.	Maximum percentage of increase in deflection due to vibration from cars.
13......	158′ 3″	0.72″	38	17′ 9″	— 27 to 40	20	2.5 to 3.7	2.5 to 4	3.6	30
14......	175′ 0″	0.41″	19	21′ 10½″	36 " 47 +	12	....	3 " 4.4	4.2	8
15......	104′ 0″	0.39″	31	17′ 4″	28 " 47 +	20	2.6 to 3.9	3 " 4	5.0	28
16......	127′ 9″	0.42″	27	18′ 3″	28 " 50 +	21	2.6 " 4.1	3 " 4	4.5	21
17......	123′ 6″	0.28″	19	20′ 7″	21 " 40 +	20	2.9 " 3.4	3 " 4 +	5.5	12
18......	106′ 0″	0 50″	39	17′ 8″	32 " 40	13	3.3 " 4.1	3.6 " 4 +	4.2	35
19......	126′ 0″	0.33″	22	21′ 0″	31 " 50 +	13	3.2 " 5.1	3.7 " 4 +	5.0	20
20......	200′ 0″	0.5″*	21	28′ 7″	— 24 " 44 +	18	1.9 " 3.6	2.5 " 3	4.2	Small
21......	147′ 6″	0.5″*	28	24′ 7″	23 " 40 +	17	2.1 " 3.3	2.5 " 3	4.3	"
22......	147′ 1½″	0.8″*	45	13′ 4½″	20 " 40 +	28	1.9 " 3.7	2.1 " 3	3.3	7
23......	146′ 6⅜″	0.6″*	34	16′ 3⅜″	23 " 52 +	26†	2.1 " 4.8	2.8 " 4	4.0	16

* Not well determined.

† Diagonal.

DISCUSSION.

H. B. SEAMAN, M. Am. Soc. C. E.—The method of making the experiments described by the author is open to the criticism that two results are compared which are not directly comparable. The records obtained by the instrument would appear to represent the actual conditions of strain, as nearly as can be ascertained in practice, but the calculations of static strains are much less reliable, as they include many indeterminate conditions, and should be replaced by actual strain diagrams. For this purpose it is necessary, in making the diagram of dynamic strain, to have in all cases a special train, and to record its position at each instant. The train should then be placed in corresponding positions, at rest, to obtain the diagram of static strain. This will be difficult to accomplish, but the results of any other method are open to serious question. Mr. Seaman.

It would also seem preferable to obtain a large number of tests upon a few selected bridges of varying spans, rather than upon a great variety of bridges, since the varying peculiarities of each bridge may affect results, which would be erroneously attributed to a change of span. Different sets of experiments, with varying designs, might be desired; but they should be kept separate so that the effect of span, or of rate of application, may be clearly traced.

It is also important that the effect of vibration or oscillation should be distinguished from that of impact or sudden application. Unbalanced drivers appear to cause the principal irregularity, and the effect is plainly noticeable when riding upon a badly balanced engine, passing rapidly over a long span.

The effort to deduce an impact formula from the centrifugal force due to the curve of deflection or from the effect of camber, is objectionable, because these conditions vary with different bridges, and, in some instances, are framed out altogether. The speaker entered into the theoretical consideration of the subject some time ago and adopted the general formula:

$$I = \frac{C}{\sqrt{L}} + C^1$$

Where I = percentage of increase of strain.

L = distance through which the moving load must pass to produce the given strain.

C and C^1 = constants.

This formula was based upon the assumption that I would vary with the time of application of the load. C^1 is the effect of oscillation, which would be assumed as approximately constant.

In outlining the paper on bridge specifications,* which the speaker

* *Transactions*, Am. Soc. C. E., vol. xli, p. 140.

Mr. Seaman. recently presented to the Society, the value of C^1 was included in the allowable live strain and believed to amount to 30% or more. The remaining values were given in the form of a table, but were slightly varied to conform to a quadrant of the curve of an ellipse. This is found to give a heavy increase for short spans, and the experiments appear to confirm this view. The maximum increase would be upon that span where the deflection would be half the total drop of the load while passing to mid-span. The span would probably be about 5 ft. in length, and the increase might approach 100 per cent. On these short spans the effect of variation in speed would be most apparent. This effect is also noticed when the engine first comes upon the bridge.

The results obtained by the author, which include those due to both impact and oscillation, are interesting and valuable, and it is hoped that they will be amplified with special reference to short spans and sudden applications of the load.

Mr. Berger. BERNT BERGER, Assoc. M. Am. Soc. C. E.—The speaker is inclined to agree with Mr. Seaman in regard to the time of application of the load, but he is not willing to go quite as far in the application of the idea. The author's experiments show that the effect of speed in causing vibration increases with the shorter spans, and Mr. Seaman insists further that if experiments were made on spans 3 to 6 ft. in length, it would be found that the additional effect due to the shortness of the span and the sudden application of the load would amount to 100%, especially with high speeds. The speaker is inclined to think that on 3 to 6-ft. spans the rails alone would support a train going at certain speeds without the assistance of the girders. There is such a thing as the necessity of time for the transmission of stress. It takes a certain time for the members of a structure to feel the application of a load. The speaker does not mean to say that he would bridge 3 to 6-ft. openings with rails alone, because a train might come to a standstill upon them.

Regarding the question of deflections, it is well known that some are apt to give it too much weight; but it is interesting to note that the deflections observed in the author's experiments agree well with the strains as calculated for the loads used. Of course, there was neither time nor opportunity for the experiments to be made on every member in the bridge, but as far as it was possible to cover the subject, the chord strains seem to agree with the deflections very well. As far as the web members are concerned, there are, as might be expected, higher percentages of increase for the strains than for the deflections; because for a maximum deflection there is a different position of the load than for a maximum strain on the web member. The experiments also bring out some interesting points in regard to the distribution of the strains on different parts of the same member. In the chords, for instance, where they consist of eye-bars, the strain seems to be quite evenly distributed, as it also is in the

hip suspenders, where they are eye-bars, and the floor is suspended. Mr. Berger. But where they are riveted and the floor beams are riveted to them in turn, it is found that in one case the outer portion carries 40% and in another case 70% of that carried by the inner portion. It would be interesting to know at what height above the floor the apparatus was applied, and whether the distribution of the strain would vary with the distance from the point of application of the load.

With all the satisfaction which can be derived from the author's experiments as to the behavior of bridges under strain, it is somewhat depressing to note that structures which are considered rigid —rigidity being the aim and object of modern bridge design—show no more freedom from vibrations than structures which the author characterizes as flexible.

Of course, it is practically impossible for one man to cover exhaustively so wide a field of investigation in one series of experiments; but the author has applied himself to his work with great care, and has given by far the most important contribution to the study of the action of bridges under moving loads. The speaker does not believe, however, that by the methods used by the author the true effect of speed alone can be said to be determined independently of other influences. He agrees with the author, however, in the conclusion that the effect of speed alone is of no practical importance, except, perhaps, for floor beams and stringers. The accidental, and more or less unavoidable, inequalities in the track, and the imperfections in the rolling stock, play a far more important part. It is very difficult to separate their effect from that of speed alone, and this effect will, of course, be subject to considerable variations with varying speeds.

T. Kennard Thomson, M. Am. Soc. C. E.—Although bridge building has been reduced to an exact science, yet for years engineers, in proportioning the parts of bridges, have been guided by two considerations: *First*, by some German experiments, which were very good in their way, but which by no means covered the ground; and, *second*, by what is politely called judgment, but which is really nothing but good guessing. With this in view, when discussing Mr. Seaman's paper,* the speaker suggested that any engineer who would make an elaborate set of experiments on bridges, as they actually are, would receive the thanks of the profession. The author has anticipated this hope by making a very good start. The speaker thinks that much more satisfactory results could be obtained by many experiments carried on under the direction of one capable man, than by the spasmodic efforts of a number of men, each of whom would have more or less divergent methods, and that the best way to accomplish this is to secure by subscription a fund to enable such a man to devote several years to this work. As the entire profession is to be benefited, each member of it should be willing to contribute. Mr. T. K. Thomson.

* "The Launhardt Formula, and Railroad Bridge Specifications." *Transactions*, Am. Soc. C. E., vol. xli, p. 198

Mr. G. H. Thomson.

G. H. Thomson, M. Am. Soc. C. E.—This paper has introduced a subject interesting to many engineers, and reminds the speaker of his own gauge studies, about fourteen years ago, of train action upon old railroad bridges of twenty-five years' service, combining the results of gauging with stress computations and examinations of the material of the old bridges after removal. From about this time he began to design railroad bridges with reference to the speed of the loading.

The sudden imposition of a load on a truss-member, with concomitant and responsively rapid lengthening, and a sudden removal of load, with like rapid shortening, may induce in some bridges more truss motion than is desirable.

The undulations noted by the author can be found in familiar examples. A rubber band, with a suspended ball, for instance; or an experiment illustrating the effect of time loading can be made with a butcher's spring scales. Place a 10-lb weight on the hopper, and the gauge registers, say, 13 lbs., pulsating back to 8 lbs., and so on, up and down, until quiescent. Place now 10 lbs. of fine shot in a funnel, controlling with the finger, the discharge of the shot from the spout so that it can be slowly delivered to the hopper; the gauge will then record slowly 1 lb., 2 lbs., and so on, up to 10 lbs., but no more.

Some bridges pulsate (much after the manner of scales), during the passage of trains and for some time afterward. Bridges are not designed for purposes of weighing, however, and bridge-members are not coiled springs.

The results shown by the author, due to the lengthening and shortening of members, are about what is to be expected from the nature of the loading, taken together with the type of trusses, length of panel, etc.; and the observed undulations will vary much as the sectioning of the members vary under the variety of specifications now in vogue. For illustrations of this, take the diagram Fig. 15, and select any bar, say *a* (to load which consumes nine seconds of time), and consider the upper eye fixed, and the lower eye movable; and further, referring to the diagram, Fig. 16, it will be seen that the lower eye is shown in its movements as follows: Line 1 is the place of the eye corresponding to a bridge upon its false works; the false works being removed, the eye will descend to line 3, which position is constant for the structure, except at such times as train service may call the bar into action, when the eye, under normal conditions of train-loading, coupled with adequate and judicious bar section, will descend to line 6; during the passage of the train, the eye will move between the limits of lines 3 and 6, with much of this movement between lines 4 and 5; and as the train leaves, and after it is entirely off the bridge, the eye movement gradually lessens, until, the truss motion ending, finds the eye at its constant position at line 3.

In case the imposition of the load is irregular in spacing and in-

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TABLE No. 6.

Location of diagonals.	Case No. I. One single moving load.					All Panels Loaded by Live Load up to Diagonal Considered and Spaced 25 Ft. = 1 Panel.														
						Case No. II.					Case No. III.			Case No. IV.			Case No. V.			
	Stretch of each diagonal.	Time of loading.	Stretch per second.	Time for release of loading.	Release* of loading per second.	Stretch of each diagonal.	Time for loading.	Stretch per second.	Time for release of load.	Release* of load per second.	$\frac{\text{Live load}}{\text{Total area}} \times \frac{1}{E} = X.$	Total stretch in inches, 360 ins. × X.	Stretch in inches per second of loading.	$\frac{\text{Live load}}{\text{Total area}} \times \frac{1}{E} = X.$	Total stretch in inches, = 360 ins. × X.	Stretch in inches per second of loading.	$\frac{\text{Live load}}{\text{Total area}} \times \frac{1}{E} = X.$	Total stretch in inches = 360 ins. × X.	Stretch in inches per second of loading.	
		Secs.	Ins.	Secs.	Ins.		Secs.	Ins.	Secs.	Ins.		Ins.	Ins.		Ins.	Ins.		Ins.	Ins.	
a.......	For all diagonals— Stretch one way = 0.12 in. " both ways = 0.24 in.	9	0.0133	1	0.1200	For all diagonals— Stretch one way = 0.12 in. " both ways = 0.24 in.	9	0.0133	9	0.0133	$\frac{1}{4450}$ L	0.0791	0.0090	$\frac{1}{4030}$ L	0.0892	0.0099	$\frac{1}{3660}$ L	0.0984	0.0109	
b.......		8	0.0150	2	0.0600		8	0.0150	9	0.0133	$\frac{1}{4470}$ L	0.0802	0.0100	$\frac{1}{3970}$ L	0.0905	0.0113	$\frac{1}{3710}$ L	0.0970	0.0121	
c.......		7	0.0172	3	0.0400		7	0.0172	9	0.0133	$\frac{1}{4400}$ L	0.0818	0.0117	$\frac{1}{3930}$ L	0.0916	0.0131	$\frac{1}{3790}$ L	0.0950	0.0135	
d.......		6	0.0200	4	0.0300		6	0.0200	9	0.0133	$\frac{1}{3950}$ L	0.0910	0.0152	$\frac{1}{3640}$ L	0.0990	0.0165	$\frac{1}{3660}$ L	0.0985	0.0164	
e.......		5	0.0240	5	0.0240		5	0.0240	9	0.0133	$\frac{1}{3180}$ L	0.1130	0.0226	$\frac{1}{3100}$ L	0.1160	0.0232	$\frac{1}{3280}$ L	0.1095	0.0218	

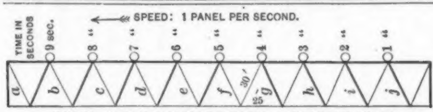

Fig. 15.

Note.—E = Modulus of elasticity = 30 000 000. *Shortening due to release.

Cases Nos. I and II.—Bridge has no weight. Diagonals stretched by the load (at 10 000 lbs. per square inch), so that the extension of each diagonal ($\frac{1}{3000}$ of length), is an equal amount.

Case No. III.—Dead and live load considered. Sectional areas of diagonals proportioned at 10 000 lbs. per square inch for live and dead loads.

Case No. IV.—As Case No. III, but sectional areas proportioned at 15 000 lbs. per square inch for dead load.

Case No. V.—As Case No. IV, but sectional areas proportioned to G. H. Thomson's specification of 1894 for the live load.

Mr. G. H. Thomson.

tensity, coupled with flat wheels, hammer blows, etc., and, in case the elastic relations inhering in the trusses are lacking in co-ordination (a fault of the design due to unsuitable length of panel, blind adherence to current specifications as to unit stresses, inattention to change of length of members, etc.), the eye will move beyond line 6 to line 7; and upon removal of the live load, sometimes, will return to line 2. This distance, lines 6 to 7, and also lines 2 to 3, can be stated as the "abnormal" travel of the eye; the distance, lines 3 to 6, the "normal"; in which "normal" may be understood to represent a measure of limitation determined by the engineer from his knowledge of the forces he handles, in reference to, and resident in, the material he uses; say its qualified powers of elastic resistance.

A bridge may be heavy, or over stocked with section, and therefore of low unit-stresses, and yet be "abnormal" in action; again, light

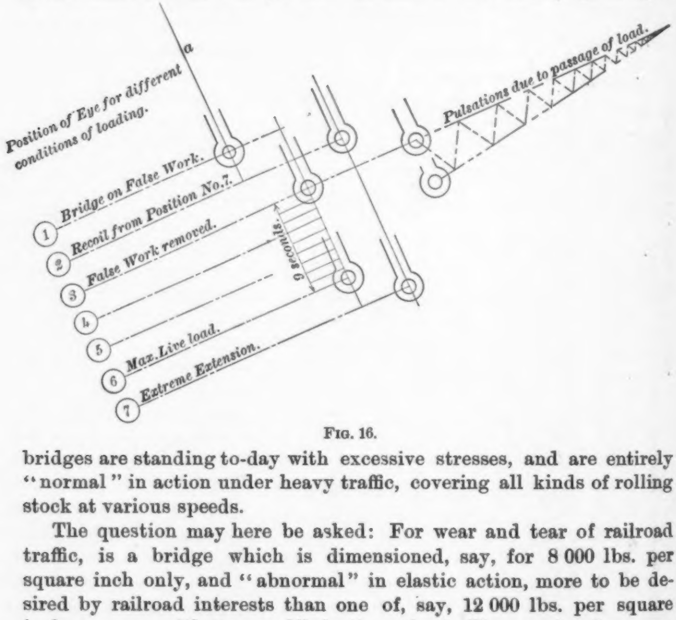

FIG. 16.

bridges are standing to-day with excessive stresses, and are entirely "normal" in action under heavy traffic, covering all kinds of rolling stock at various speeds.

The question may here be asked: For wear and tear of railroad traffic, is a bridge which is dimensioned, say, for 8 000 lbs. per square inch only, and "abnormal" in elastic action, more to be desired by railroad interests than one of, say, 12 000 lbs. per square inch, or more, with "normal" elastic action? The speaker has some very decided views on this point.

The speed of trains enters largely into matters "abnormal." Viewed briefly, there is a quantitative triad, *viz.*, train weight, train space and train time, as cause; and producing, as effect, extension, which is a manifestation of the totality of molecular action. Between cause and effect there is a relation of transmission.

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In a bar there is, in a given unit of time, the long spaces of train weight and the short bar spaces, the result of molecular motion. The ratio between the train space and bar-extension space involves the matter of "normal" or "abnormal" truss action.

Whatever may be the order of arrangement of the grouping of the elemental units of mass in a bar; whatever may be their mode of assembly; whatever may be the theory of their mutual coherence; whatever may be the quality of their being and action, we are logically justified in assuming that all these relations obtain under a rule of existence, *i. e.*, a physical law of coherence of association of units, constant in its operation, within the limits of the scope of the law. Any disturbance of this coherence of the elemental units of mass involving the change of interaction between the units, amounting to an infringement of this law, finds a penalty in compromised molecular integrity. The physical injury to the bar a, due to extension from line 1 to line 2 (occurring but once. and being very small), can be disregarded. The extension from line 3 to line 6 can be kept within the limits of the provisions of the law.

One of the products of the changed inter-action of the elementary units of mass (say, of a tension bar), finds its expression in the phenomenon of elongation, the magnitude of which can be visualized and measured, if of appreciable amount. If it is desired to call into service the force stored up in a mass, and therein resident by virtue of the law of coherent association, keeping within the outside limit of the law, the units must be given plenty of time to adjust themselves to the changed inter-action; or rather, it is not so much a matter of time, or of how fast the loading, or how slow the stretching of the bar, as of the relation of time between the loading and stretching.

(1) Train action	is to be met by	molecular	reaction.
(2) Train spacing	" "	"	spacing.
(3) Train motion	" "	"	motion.
(4) Train momentum	" "	structural	momentum.
(5) External relations	"	internal	relations.

That the ratio of space covered by metallic linear motion to space covered by train motion varies much in different parts of a truss, is shown on the diagram, Fig. 15, where uniform loads are used. The diagram is merely for an illustration.

Case I.—All diagonals under the live load have an extension of 0.12 in. The extensions per second range from 0.0133 in. for bar a to 0.024 in. for bar e, a difference of 0.0107 in. The shortening of the bars a and e per second ranges from 0.12 in. to 0.024 in. Bar a receives its severest test in unloading.

Case II.—In this case the extensions are the same as in Case I, save in the shortening due to releases of load which $= 0.0133$ in. per second for each bar.

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Case III.—In this case dead as well as live load is considered at 10 000 lbs. per square inch, and is fairly representative of many bridges in service. The extensions range from 0.0090 in. for bar *a*, to 0.0226 in. for bar *e*, per second. These extensions are computed, but when gauged, another set of extensions, depending upon the speed of the trains, will obtain.

Case IV.—With the dead load at 15 000 lbs. per square inch, and live load at 10 000 lbs. per square inch, the extensions gauge nearer to computation than in Case III.

Case V.—The extensions gauge and compute the same. Flat wheels, hammer blows, unequal loading and spacing have no measurable effect. There are many bridges of from 125 to 240-ft. span, built in accordance with these specifications. Some 200-ft. spans are of narrow depth; all are free from undue structural motion.

Taking up now Case III, the stretch of the whole bar, *a*, in one second of time = 0.0090 in., the bar being 30 ft. long. The stretch of 1 ft. of bar for one second of time $= \frac{0.0090 \text{ in.}}{30} = 0.00030$ in. While the bar is extending this amount for 1 ft. of its length, the train has passed 25 ft. or $\frac{0.00030 \text{ in.}}{25} = 0.000012$ in. = the extension of 1 ft. of bar while the train moves 1 ft.; and $\left(\frac{0.000012}{12.000000}\right) = \frac{1}{1\,000\,000}$, that is, the train moves 1 000 000 times as fast as the movement of metal in the bar.

First.—Comparison of Case III with Case V. $\left(\frac{\text{Metal motion}}{\text{Train motion}}\right) = R$. Speed = 25 ft. per second.

	Range.
Case III.—Bar *a*	1 000 000
" *V.*— " *a*	882 352
" *III*— " *e*	387 161
" *V.*— " *e*	416 236

Case III.—Bar *a*	1 000 000	
" " — " *e*	387 161	
	613 839	= Difference of range between *a* and *e*.
Case V.—Bar *a*	882 352	
" " — " *e*	416 236	
	466 116	= Difference of range between *a* and *e*.
Case III.—Bar *a* =	1 000 000	
" *V.*— " *a* =	882 352	
	117 648	= Difference in range.
Case III.—Bar *e* =	387 161	
" *V.*— " *e* =	416 236	
	29 075	= Difference of range.

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Case V absorbs train motion faster into bar a.
" " " slower into bar b.

Suppose, now, that for Cases III and V the speed be increased threefold, or to 75 ft. per second. For Case III the same range will obtain for all speeds, by computation, but it does not follow that the gauged extensions for the high speed will be the same as for the low speed.

For Case V the live loads (1894 specifications) are found by the formula for permissible unit stress $= F\ (\ T - l)\ a$
in which $F =$ general factor $= 7\ 500$ lbs,
$T =$ time for imposition of load in seconds,
$a =$ ratio factor $= 500$.

Hence for bar a, at 9 seconds, $P = 9 \times 500 = 4\ 500 + 7\ 500 = 12\ 000$ lbs. per square inch.

And for bar e, at 5 seconds, $P = 5 \times 500 = 2\ 500 + 7\ 500 = 10\ 000$ lbs. per square inch.

Second.—Comparison of Case III with Case V at high speed $= 75$ ft. per second.

	Range.
Case III.—Bar $a =$	1 000 000
" *V.*— " $a =$	977 900
" *III.*— " $e =$	387 161
" *V.*— " $e =$	556 586

Case III.—Bar $a =$	1 000 000	
" " " $e =$	387 161	
	613 839	= Difference between a and e.

Case V.—Bar $a =$	977 900	
" " " $e =$	556 586	
	421 414	= Difference between a and e.

Case III.—Bar $a =$	1 000 000	
" *V.*— " $a =$	977 900	
	22 100	= Difference between a and e.

Case III.—Bar $e =$	387 161	
" *V.*— " $e =$	556 586	
	169 425	= Difference between a and e.

Third.—Comparison between Cases III and V:
Difference of range between a and e

at 25 ft. per second	613 839	466 116 = 147 723
" 75 " "	613 839	421 414 = 192 425
	000 000	24 702

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By adjusting the speed or rate of metal motion, making it slower in certain members and faster in others, co-ordinated elastic relations in concordance with the assumed stresses result.

A number of covered, through Howe bridges, of thirty years' use, came under the speaker's notice, wherein the rods, three at each panel point, and of the same size throughout the bridge (with the actual dead load and a live load of 2 000 lbs. per lineal foot), stresses were

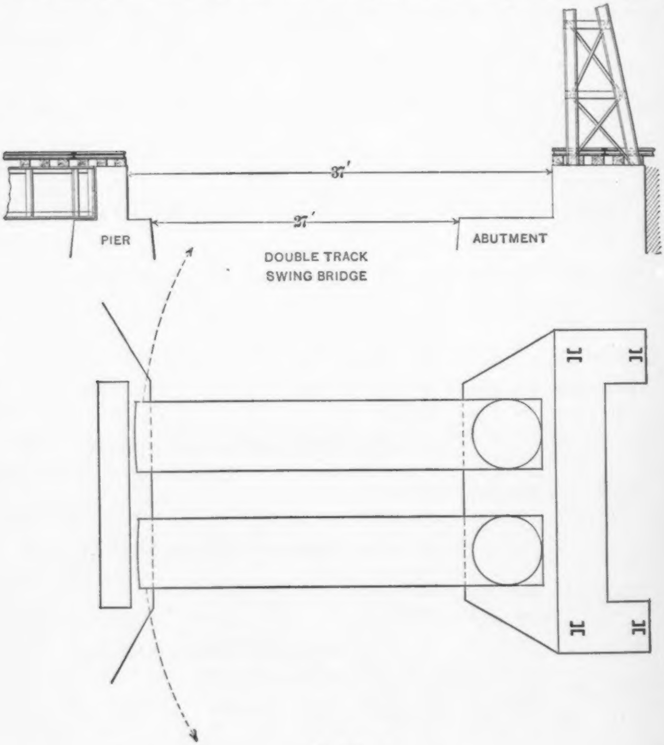

FIG. 17.

found of 30 000 lbs. per square inch for the end rods, and 15 000 lbs. per square inch for the center rods. These old bridges furnish a record of persistence of structure due to the relation of metal motion to train motion. The non-failure of the end rods at stresses beyond the elastic limit can be explained by the fact that the motion induced in the metal was a matter of the slow delivery of the

PLATE XI.
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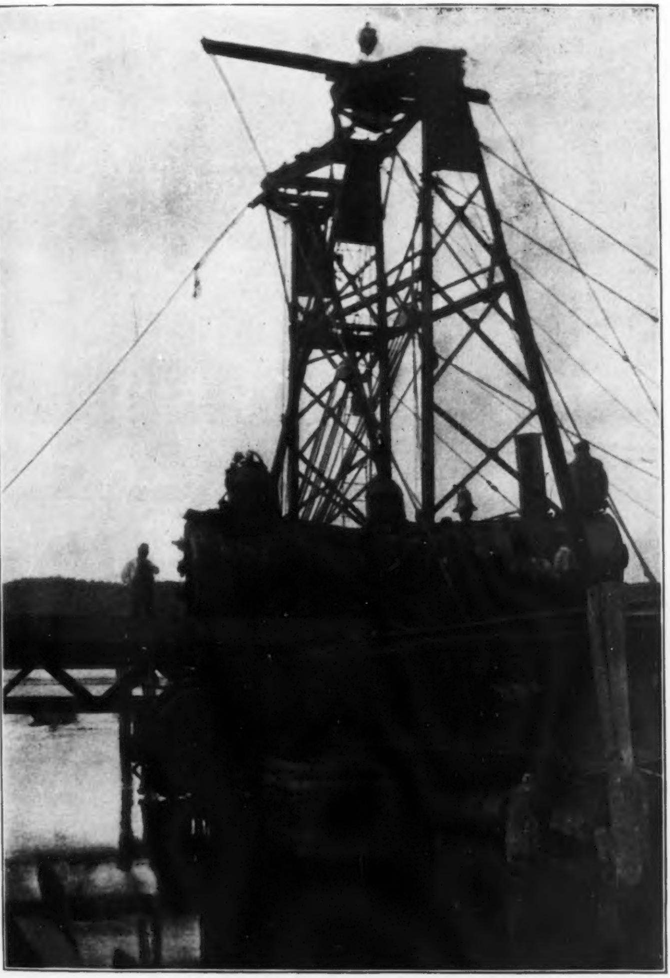
THOMSON ON TRAIN-LOAD EXPERIMENTS.



train-load to the rod. The end rods of such old bridges often show better fractures than the center rods. Tests of old rods often give high ultimates and elastic limits, about the same for center as for end rods, but the center rods break more like " pipe stems " than the end rods of the same bridge; or, a rod will withstand a live load for a maximum number of applications, if the load is slowly imposed. If a rod has already a constant or dead-load stress, still, the live load, if imposed slowly, will be better resisted by the rod, *i. e.*, if the speed of the metal motion is favorable when compared with the speed of the loading. Mr. G. H. Thomson.

The great length of the rods of "through" Howe bridges, doubtless, is favorable for the reception of the loading.

The author appears to attach considerable weight to the matter of deflection. Viewed as a factor involving structural integrity of truss bridges, the magnitude of the central deflection may be of value; but with plate-girder bridges the speaker does not think the fact of deflection is of much value. With short-span deck-plate girders of light section, the amount of the deflection at high speed is frequently one-half that obtained by low speeds or with engines at rest. A rapid passing of the engine over a light plate-girder reminds one of skating over thin ice. The speaker recalls a case where a canal boat, knocking a track stringer out of a bridge, the bridge was used for several days without any stringer whatever. This was a panel of 10 ft. In another case a deck-girder of 10-ft. span had been removed (leaving the rails and ties in place), preparatory to substituting a new bridge. A belated fast train, paying no attention to signals, ran over the opening, on the rails, *i. e.*, with no bridge to sustain the train. In yet another case, which is illustrated by the diagram, Fig. 17, a double-track opening of 37 ft. was covered by two deck-lattice girders, one for each track. The girders were turned upon a center at the abutment end, while the pier end was sustained by guys held from a tower resting upon the abutment. The girders, for one track, were in position for traffic, while for the other track the girders were " swaying " 90° from the track, leaving a clear opening of 37 ft. An express train approaching applied the brakes at a point 500 ft. from the draw. The engine jumped the opening as shown by the photograph, Plate XI, the forward truck clearing the opening entirely. Also, in trusses, some counters have been observed in which the last wheels of a passenger train produced twice the extension made by the engine drivers.

Regarding vertical truss motion ("jumping" in particular, which is most easily felt on through-bridges while standing thereon during the passage of a fast and light train—this is not to be confounded with the vertical motion of mere deflection), the web members, principally, govern its extent and character. Some time is consumed to absorb the train weight into the chords, and hence they are not given to the over-

Mr. G. H. Thomson.

extension lines 6 to 7 of the diagram, Fig. 16. If a train passes two panels in, say, one second, the extension of the diagonal is measured by y. Resolved into z it is not easily appreciated; into x it is visible. For the two panels of a diagonal system the vertical motion $= 2\,x$, of which there is one x for each diagonal (see diagram, Fig. 18); and $2\,x$ for diagonals, and one x for the vertical post (see Pratt system, of diagram, Fig. 19).

The term "structural motion" refers to the motions induced in structures by the moving loads.

Structures without some motion are inconceivable, but undue structural motion may be controlled and reduced to reasonable limits without increasing the cost.

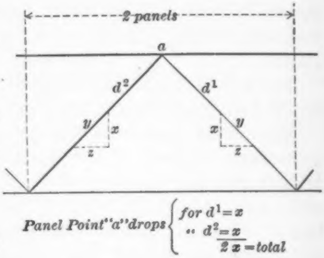


FIG. 18.

A bridge, in strength, must meet all statical conditions; but is this enough? The fractures, etc., of the metal of old bridges, with motion, do not compare favorably with those old bridges that were free of motion.

To make bridges free of motion, a number of points require recognition: The type of truss, the length of panel (with its value to receive equally and distribute or disperse the loading), the elastic resistances of material, the positive momentum of trains with negative momentum of bars, etc.,—all of these co-ordinated to time ratios, *i. e.*, speed of trains—or, train motion, *versus* molecular motion.

A structure free of motion, void of over-strain, is suggestive of permanence and persistence, and a railroad bridge that is strong, as viewed from a statical point, if free of structural motion, invites confidence and holds it. Many bridges having spans of from 125 to 240 ft. of Class V specification, are in service, where the deflection is only about $\frac{1}{7000}$ of the span, though this ratio in itself is of no particular value.

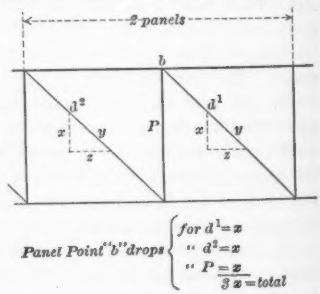


FIG. 19.

It is not necessary to put so much metal in members that consume much time to load, *i. e.*, the bottom chords. A through-bridge designed August 15th, 1894, under Class V specifications (not built), for Augustus Mordecai, M. Am. Soc. C. E., for Erie engines, weighed, as per detail plans, 123 tons for a clear span of 150 ft.

CORRESPONDENCE.

J. B. Johnson, M. Am. Soc. C. E.—The author deserves, and will doubtless receive, the thanks of the profession for his remarkably thorough experiments and his exhaustive study of the same. While his results had all been anticipated and provided for in American bridge practice, yet there was no positive proof that the assumed limits of impact effect were very nearly correct. Mr. Johnson.

Many engineers have had a lurking fear that there might be immediate or cumulative stresses far beyond those ordinarily provided for, and these experiments will reassure them that the present usages in the United States, as to working loads, are fully justified.

The discovery of certain large secondary stresses in plate girders, also, when eccentrically loaded, as on shelf angles, is only what an analysis of such cases would have shown. This fact simply emphasizes the necessity for the complete analysis of engineering structures, in all their parts and details, for the actual conditions of loading, which is now receiving attention in the leading engineering schools, as well as in the computing offices of practitioners. It is evident that the limit of fruitful mathematical analysis in engineering design has not yet been reached.

F. E. Turneaure, Assoc. Am. Soc. C. E.—Mr. Seaman's criticism in regard to comparing the observed results with the computed, would appear to be hardly pertinent. Computed results are used only in the discussion relating to the effect of speed alone, and then in but three out of the eight bridges so discussed are the computed results of any significance. In the other five cases a special train was used, and was moved over the bridge at so low a speed as to give a strain diagram unaffected by dynamic action, and which could therefore serve as a standard. For convenience, however, all stresses were referred to the computed values, and the significance of the diagrams in Figs. 12, 13 and 14 is not in the absolute values of any of the ratios, but in the horizontality of the lines, showing that the deflections and stresses were practically the same at all speeds. Whether the stresses of the various diagrams from this special train coincided or not with the computed values, is of no consequence, as the observed values are themselves directly comparable. Mr. Turneaure.

The purpose of the theoretical discussion of the effect of speed of application was, not to derive a working value for it, but to show that for ordinary spans it could be of no practical consequence. Camber may of course remove it entirely, or, in fact, give it a negative value, all of which is quite immaterial in practice. The author aimed to show that, both theoretically and experimentally, practically all the increase in deflection, or stress, in the spans tested could be attributed to vibration and oscillation. If in Mr. Seaman's formula it is intended to include the effect of speed of application (that which would still remain were the load perfectly balanced) in the first term, and

Mr. Turneaure. all other effects in the second term, in the author's opinion the first term should be made equal to zero, at least for all spans above 25 ft. in length, and the second term made to vary inversely with span length. In very short spans, such as 5 ft., the effect of speed would theoretically be great if the beams were shallow, but would be a maximum, not at the center, but some distance beyond. In a 5-ft. span for example, with a depth of beam equal to one-tenth the span, there would be, according to Zimmerman's formula, an increase of 30% in the central deflection, and more than 100% for a point two-thirds across the beam. For still shorter spans the central deflection becomes less than for static load, but near the abutment it would be many times increased. All of this assumes absolutely rigid abutments, track and rolling load, but the deflections involved in such very short spans are so small that a very slight yielding would entirely vitiate any calculations such as the above. Cases of trains passing safely over very short spans with no other support than the rails, as mentioned by Mr. G. H. Thomson, are instances to the point, but the author ventures to say that in such cases the abutment, or other "landing place," received a pretty severe pounding. If the load is constrained to move in a straight line there could be no effect of speed either in adding to the load or in reducing it. In the actual case, both girder and supports are elastic. The effect of deflection of the girder is to increase the pressure, while the effect of elastic supports is to diminish it. In the author's opinion the former is too small to take into account on any but the very shortest spans, and in those there would doubtless be sufficient elasticity in the supports to more than counterbalance this effect. The practice of supporting short girders on wooden wall plates would seem to be a good one, as tending to relieve both girder and masonry.

A matter which should unquestionably be taken account of, and which is clearly brought out by both deflection and stress diagrams, is the increase of stress in short girders caused by the locomotive, and which increases rapidly with decrease of span.

Answering Mr. Berger's question in regard to the stresses in riveted hip verticals, the author would say that the stresses were determined at points close to the floor beams. Time was not available for a more extended series of experiments along this line.

In regard to the effect of the vibration, or "jumping," on truss members, the experiments do not bear out Mr. G. H. Thomson's statement that such effect is mostly in the web members, and that the chords receive their stress too slowly to be much affected thereby. While the horizontal component of the web distortion may not be readily appreciated by an observer standing on the vibrating truss, the variation in chord stress with truss vibration is made very evident by watching the action of the extensometers at such times. The close agreement in the relative variation in chord stress, and vibration of the entire truss, in a large number of tests, would appear to be a good indication of the real state of things.